The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2012–2013**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5, only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below.

(a) A pyramid has base b cubits, height h cubits, seqt s hands per cubit and angle of slope θ. Write s in terms of b and h, *and* in terms of θ. Who might have worked with seqts and why? A pyramid has base 140 cubits, and seqt 5 hands 1 finger per cubit, where 4 fingers equal a hand. Find its height and, to the nearest *degree*, its angle of slope.
(7 marks)

(b) Describe the *structure* of **Book I** of Euclid's *Elements*, giving illustrative examples.
(7 marks)

(c) The sentence below refers to *Scipione del Ferro* of Bologna University.

Around 1515, he found algebraic solutions to a wide class of cubic equations, but kept his discovery quiet, confiding it only to a pupil, Antonio Fior, and his son-in-law and successor at Bologna, Annibale della Nave.

Explain the terms *algebraic* and *a wide class of cubic equations* used here. Why might *del Ferro* have kept his discovery secret? Describe the roles played by *Fior* and *della Nave* in the history of cubic equations. ***(7 marks)***

(d) For what purpose did Greek mathematicians employ the *method of exhaustion*? State the principle underpinning its application and make some general comments on *proof by exhaustion*. Quote a result in the *Elements* which is proved by means of it. ***(7 marks)***

2 Where is the Babylonian clay tablet **Plimpton 322** currently exhibited? Why is it thought to be the *right-hand* part of a larger tablet? What script does it bear? How many rows and columns of *numbers* appear on it? Define a *regular sexagesimal.* ***(5 marks)***

Express the first *three* entries in each row of the tablet (the *eleventh* excepted) in terms of two *regular* sexagesimals p and q $(p > q)$, and relate them to an integer-sided triangle ABC having a right angle at C and smallest angle A. ***(5 marks)***

The *second* and *third* entries on the *sixth* row of the tablet are 319 and 481. Given that the *first*, in sexagesimal notation, is $1\,;\, n,\, 6, 41, 40$, determine the *natural number* n. ***(6 marks)***

3 State *the three classical problems of antiquity.* ***(3 marks)***

(a) Name a curve studied by the Greeks that could be used to solve *two* of the problems. Define it and say which *two* of the problems it could solve. ***(4 marks)***

(b) Define the *conic sections* in terms intelligible to a Greek mathematician and explain how they *first* entered mathematics. ***(4 marks)***

(c) Consider a square of side $2a$, its circumcircle, and the four semicircular arcs described on its sides and external to it. Write down the radius of this circumcircle. Show that the area of a *minor* segment of the circle cut off by a side of the square is $\frac{1}{2}\,(\pi - 2)\,a^2$. Deduce that a *lune* bounded by the circle and one of the semicircular arcs can be *squared* . Which Greek mathematician squared such lunes and for what purpose? ***(5 marks)***

4 Outline Robert Recorde's role in sixteenth-century British mathematics. To what do you attribute his success as an author? ***(8 marks)***

Comment briefly on each of the following extracts (not all in the course), taken from Recorde's books, and name its host book.

(a) Copernicus has renewed an old theory that the earth not only moves about its axis, but is 38 hundred thousand miles from the centre of the world.

(b) . . . a paire of parallels of one lengthe, bicause noe 2.thynges can be moare equalle.

(c) *To divide an angle into ii equal partes*: Open your compasse as large as you can . . . .

(d) But there are more benefits to arithmetic. Why are accountants so well rewarded, geometers so well respected, and astronomers so knowledgeable? It is because by numbers, they have made discoveries, impossible without them. ***(8 marks)***

5 Summarize Fermat's contribution to the calculus. Why is he *not* considered one of its *inventors*? ***(6 marks)***

Let C be the curve $y = x^3$ $(x > 0)$ and let $a > 0$. How would Fermat have shown that:

(a) the length of the subtangent to C at the point (a, a^3) is $\frac{1}{3}\,a$; ***(5 marks)***

(b) the area under C and above the x-axis from 0 to a is $\frac{1}{4}\,a^4$? ***(5 marks)***

End of Question Paper