The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2012–2013**

Time Series **2 hours**

Marks will be awarded for your best ***three*** *answers.*

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (but no textbooks) plus a calculator that conforms to University regulations.
There are 99 marks available on the paper.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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1 The plot below shows quarterly total sales (in thousands) of one-day-old turkey chicks from hatcheries in Eire (source Pole, A., West, M., and Harrison, P.J., 1994, Applied Bayesian Forecasting and Time Series Analysis, Chapman-Hall).

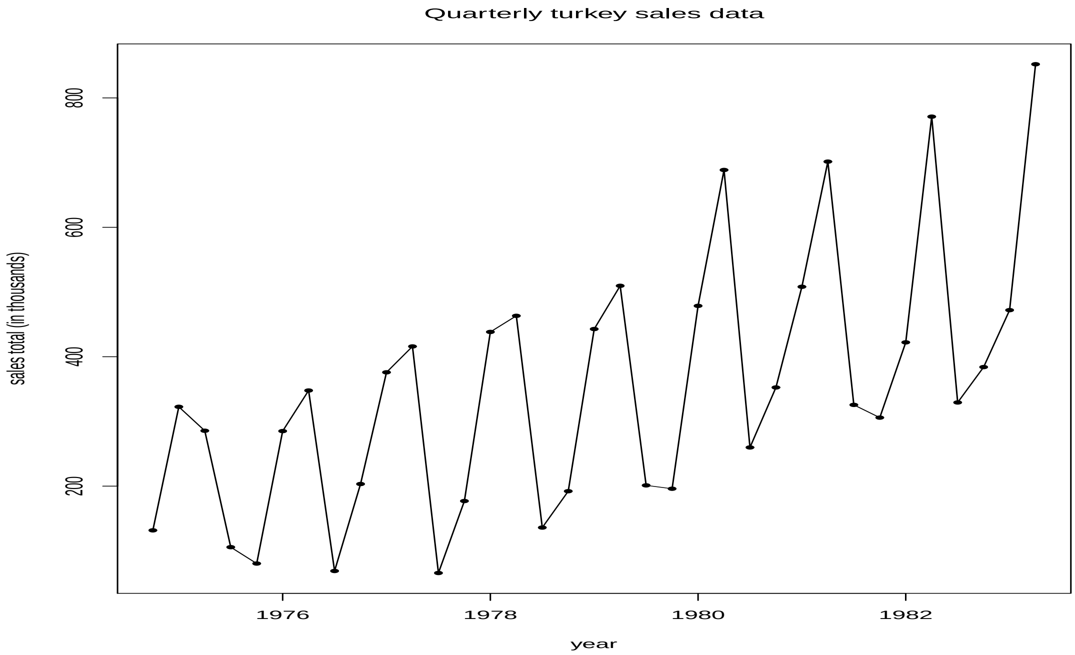

Figure 1: Quarterly sales turkey chicks data

(i) Briefly describe the features of the data. ***(4 marks)***

(ii) Suggest a transformation of the time series data y_t, likely to result in a stationary time series x_t and write down x_t as a function of y_t using appropriate differencing notation. ***(4 marks)***

(iii) For a time series x_t (of length 31) derived from y_t by a suitable transformation, the sample ACF and the sample PACF are tabulated below:

Lag	1	2	3	4	5	6	7	8
ACF	r_1	r_2	-0.468	0.700	-0.397	0.134	0.051	0.002

and

Lag	1	2	3	4	5	6	7	8
PACF	-0.650	-0.488	-0.832	-0.565	-0.300	0.124	0.090	0.032

(a) Find the values of r_1 and r_2. ***(5 marks)***

(b) Test whether x_t is a white noise. ***(5 marks)***

(c) Test whether x_t is consistent with autoregressive models. ***(5 marks)***

(d) Test whether x_t is consistent with moving average models. ***(7 marks)***

(e) Based on your analysis above, suggest a time series model for x_t that is likely to perform well when fitted to the data. ***(3 marks)***

2 Consider the time series model

$$y_t = 19 - \frac{1}{3}y_{t-1} - \frac{1}{4}y_{t-2} + \epsilon_t - \frac{1}{2}\epsilon_{t-1},$$

where ϵ_t is white noise with variance 8.

(i) Write down this model using the Backward shift operator B. ***(4 marks)***

(ii) Show that this model is causal and invertible. ***(8 marks)***

(iii) Find the mean of y_t. ***(5 marks)***

(iv) Find the variance of y_t. ***(16 marks)***

3 The plot below relates y_t the quarterly change in a company's sales to x_t the quarterly change of a market sales indicator variable.

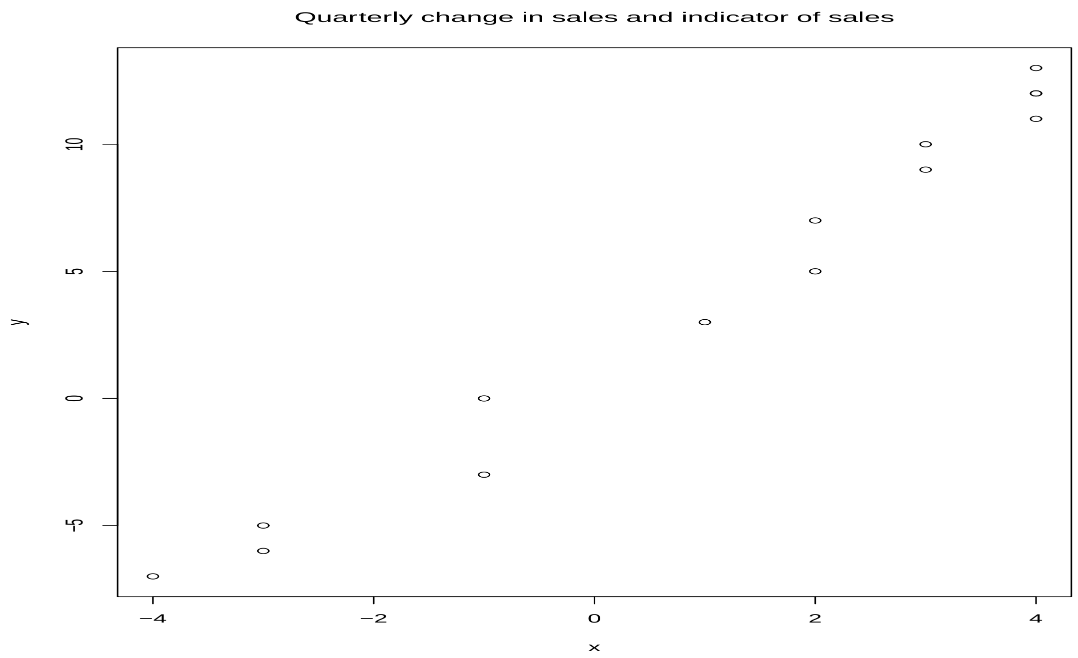

Figure 2: Relation of change of sales y_t and indicator of sales x_t

As a first model it is suggested to regress y_t on x_t, or

$$y_t = x_t\beta + \epsilon_t,$$

where β is a regression coefficient and ϵ_t is a white noise with variance 1.

However, the statistician of the company argues this model is not appropriate to model the data set. To back her argument she has provided the autocorrelation functions of the time series x_t and y_t, given in the plot overleaf (Figure 3).

(i) Explain why the statistician believes the model above is inappropriate.
(2 marks)

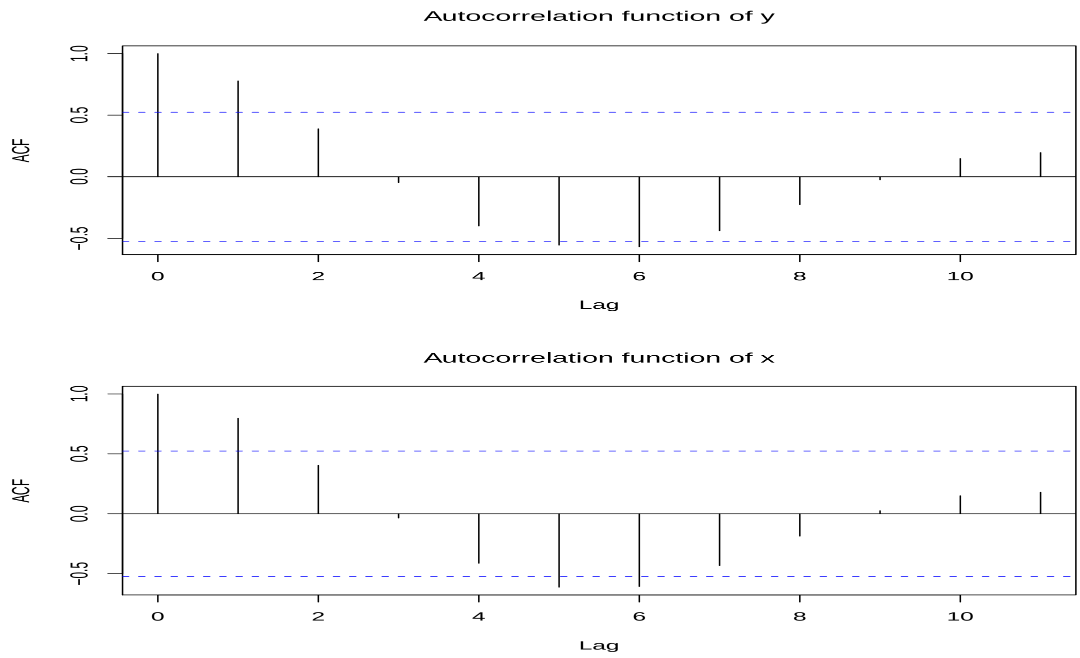

Figure 3: ACF of the time series x_t and y_t

3 (continued)

(ii) The statistician suggested a second model as alternative, given by

$$y_t = x_t \beta_t + \epsilon_t \quad \text{and} \quad \beta_t = \beta_{t-1} + \zeta_t, \tag{1}$$

where ϵ_t is as before and ζ_t is a white noise with variance 10.

(a) Give the name of model (1). ***(1 mark)***

(b) For model (1) show that $P_{t|t}$ the posterior variance of β_t satisfies

$$\frac{1}{P_{t|t}} = \frac{1}{P_{t-1|t-1} + 10} + x_t^2.$$

(10 marks)

(c) If $x_1 = 4$, $x_2 = 4$, $y_1 = 12$, $y_2 = 11$ and the prior of β_0 is $\beta_0 \sim N(2, 0.81)$, then use the result in (b) to calculate the posterior means $\hat{\beta}_{1|1}$, $\hat{\beta}_{2|2}$ and the posterior variances $P_{1|1}$, $P_{2|2}$. ***(8 marks)***

(d) If $x_3 = 3$, use (c) to obtain the one-step forecast mean of $y_3 = 9$ and the associated residual. Comment on the quality of this forecast. ***(3 marks)***

3 (continued)

(e) If instead of $\beta_0 \sim N(2, 0.81)$ the prior distribution of β_0 is set to either of the following:

(α) $\beta_0 \sim N(10, 0.81)$ or

(β) $\beta_0 \sim N(2, 100)$,

comment on whether you expect an improvement on forecasting and the general model performance for all y_t. ***(6 marks)***

(f) Suggest how the model performance can be improved. ***(3 marks)***

4 A company sets up a time series model for the yield y_t of an investment at time t as

$$y_t = x_t \theta_t + \alpha y_{t-1} + \varepsilon_t,$$

where $\theta_t = \theta_{t-1} + \eta_t$ and $\varepsilon_t = 0.9\varepsilon_{t-1} + \nu_t$. Here, x_t is a time-varying covariate, θ_t and α are regression and autoregression coefficients and the innovations η_t and ν_t are assumed to be independent, each following a white noise process. It is further assumed that η_t is independent of α, for all t.

(i) Based on the above model description, give a name for the model of y_t. ***(2 marks)***

(ii) The state vector is defined as follows:

$$\beta_t = \begin{bmatrix} \theta_t \\ \alpha \\ \varepsilon_t \end{bmatrix}.$$

Use it to express y_t in state space form, i.e.

$$y_t = \psi_t \beta_t + \kappa_t \quad \text{and} \quad \beta_t = F\beta_{t-1} + \zeta_t,$$

hence determine ψ_t, F and define the innovations of the model κ_t and ζ_t. ***(8 marks)***

(iii) Suppose this model was fitted to data $y_1, \ldots, y_n$. With information $y_{1:n} = \{y_1, \ldots, y_n\}$, suppose that the posterior distributions of θ_n and α were

$$\theta_n \mid y_{1:n} \sim N(1, 10) \quad \text{and} \quad \alpha \mid y_{1:n} \sim N(1, 2).$$

With information $y_{1:n}$,

(a) show that the 1-step ahead forecast mean $\hat{y}_{n+1|n}$ of y_{n+1} is

$$\hat{y}_{n+1|n} = y_n + x_{n+1};$$

(9 marks)

(b) show that the 2-step ahead forecast mean $\hat{y}_{n+2|n}$ of y_{n+2} is

$$\hat{y}_{n+2|n} = y_n + x_{n+2} - x_{n+1}.$$

(14 marks)

End of Question Paper