The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2012-2013**

Mathematical Methods for Statistics **2 hours**

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.
Candidates should attempt ***ALL*** *questions.*
The paper will be marked out of 80 and the allocation of marks is shown in brackets.

1 Find the first derivatives of the following functions with respect to x:

(i) $f(x) = x^2 - x^{-2}$

(ii) $g(x) = \cos(x)\ln(x)$

(iii) $h(x) = \tan(e^{3x+1})$ **(10 marks)**

2 Let c be a non-zero real number, and let $f(x, y) = x^2 + cy^2$.

(i) Show that $f(x, y)$ has exactly one critical point p.

(ii) For what values of c is the critical point p a local maximum/minimum/saddle?

(10 marks)

3 Find:

(i)

$$\int x^3 \ln x \, \mathrm{dx}$$

(ii)

$$\int_1^3 \left(1 + \frac{1}{x^2}\right)^{-\frac{1}{2}} \mathrm{dx}$$

(10 marks)

4 The set of (x, y) satisfying the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

determines an ellipse $\mathcal{E}$ in the xy-plane. Determine the area of region bounded by the ellipse $\mathcal{E}$. **(10 marks)**

Hint: start with the change of variables $x = au$, $y = bv$.

5 Let

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \text{ and } A = \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix}$$

Find

(i) $\mathbf{y}.\mathbf{x}$

(ii) $|\mathbf{x}|$

(iii) $A^{-1}\mathbf{x}$

(iv) $\mathbf{y}^T A$ **(10 marks)**

6 Use Gaussian elimination to solve the following system of equations:

$$\begin{aligned} 2y + z &= -8; \\ x - 2y - 3z &= 0; \\ -x + y + 2z &= 3. \end{aligned}$$

(10 marks)

7 Find a basis for the subspace of $\mathbb{R}^3$ that contains the following vectors:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 5 \\ -3 \\ 6 \end{pmatrix}.$$

(10 marks)

8 Let $Q(x, y, z) = 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz$. Show that $Q(x, y, z) \geq 0$ for every $(x, y, z) \in \mathbb{R}^3$.

Hint: What can you say about positive definite quadratic forms? **(10 marks)**

End of Question Paper