



The
University
Of
Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS

May 2013

Probability and Probability Distributions

2 hours

RESTRICTED OPEN BOOK EXAMINATION.

Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.

*Candidates should attempt **ALL** questions.*

The maximum marks for the various parts of the questions are indicated.

The paper will be marked out of 80.

**Please leave this exam paper on your desk
Do not remove it from the hall**

Registration number from U-Card (9 digits)
to be completed by student

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- 1** Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k(4-x) & 0 \leq x \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the value of k . *(4 marks)*
- (b) Find $P(X > 2)$. *(3 marks)*
- (c) Find $E(X)$ and $\text{Var}(X)$. *(6 marks)*
- (d) Let $Y = X^2$. Find the probability density function of Y . *(5 marks)*
- 2**
- (a) (i) Let $X \sim Bi(12, 0.3)$. Using tables, find $P(X \leq 4)$. *(2 marks)*
- (ii) Let $Y \sim Bi(12, 0.7)$. Using tables, find $P(Y \leq 8)$. *(2 marks)*
- (b) Let Z have a chi-squared distribution with 11 degrees of freedom.
- (i) Give the value z such that $P(Z \leq z) = 0.95$. *(2 marks)*
- (ii) Give bounds for $P(Z \leq 4.1)$. *(2 marks)*
- 3** A fair six-sided die is rolled twice. Assume the rolls are independent of each other.
- (a) Describe an appropriate sample space to use for this experiment. *(3 marks)*
- (b) Let E be the event that the total shown on the two dice is even, and let D be the event that the values shown on the two dice are the same. Find the probabilities
- (i) $P(E)$
- (ii) $P(D)$
- (iii) $P(E \cap D)$
- (iv) $P(D|E)$. *(8 marks)*
- (c) Are the events D and E in (b) independent? Give a reason for your answer. *(3 marks)*

- 4 Let T be the region defined by $T = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1, x \geq y\}$, and let X and Y be random variables with joint probability density function given by

$$f_{X,Y} = \begin{cases} 2 & (x, y) \in T \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the marginal probability density functions of X and Y . **(4 marks)**
- (b) Find the conditional probability density function of Y given $X = x$ (for $0 < x < 1$). **(4 marks)**
- (c) Find the conditional expectation of Y given $X = x$ (for $0 < x < 1$). **(4 marks)**
- (d) Find $P(X + Y \geq 1)$. **(4 marks)**
- 5 The number of customers who visit a shop on any day is believed to follow a distribution with mean 20 and variance 10, and the numbers who visit on different days are believed to be independent. Assuming that these beliefs are correct, find an approximate probability that the number who visit the shop in a 40 day period is at least 785. **(8 marks)**
- 6 A six-sided dice is known to be either dice A, which is fair, or dice B, which gives 1 and 2 each with probability 0.4 and 3, 4, 5 and 6 each with probability 0.05.
- (a) The dice is rolled ten times. What is the probability that 1 is rolled four times, 2 four times, 3 once and 6 once
- (i) if the dice is dice A?
- (ii) if the dice is dice B? **(4 marks)**
- (b) Given that in the ten rolls 1 is rolled four times, 2 four times, 3 once and 6 once, calculate the posterior probability that the dice is dice A
- (i) if the two dice were *a priori* equally likely;
- (ii) if the prior probability of the dice being dice A was 0.9. **(8 marks)**
- (c) With the same combination of the ten rolls as in (b), what value of the prior probability of dice A gives exactly equal posterior probabilities of dice A and B? **(4 marks)**

End of Question Paper