The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2013-2014**

FOUNDATION YEAR MATHEMATICS 1 **3 hours**

Attempt ***all*** *questions. The allocation of marks is shown in brackets. Total marks: 100.*

1 Differentiate $f(x) = 9 + x^2$ from first principles. ***(4 marks)***

2 Differentiate the following with respect to x.

(i) $y = 7x^8 - 11\sqrt[11]{x} - \dfrac{5}{x^{\frac{7}{5}}} - 10;$

(ii) $y = \left(8x^3 - 3x\right) e^x;$

(iii) $y = \tan^{-1}(3x)\,;$

(iv) $y = \dfrac{\ln x}{\cos^{-1} x}.$ ***(10 marks)***

You are expected to simplify each of your answers as much as possible.

3 Find the following indefinite integrals.

(i) $\displaystyle\int \left(x^5 - \frac{7}{x^6} + 9\sqrt[3]{x} + e\right) dx;$

(ii) $\displaystyle\int \frac{3x^2 + 1}{1 + (x^3 + x)^2}\, dx;$

(iii) $\displaystyle\int (2x - 5)\sin x\, dx\,.$ ***(10 marks)***

4 Evaluate the following definite integrals.

(i) $\displaystyle\int_1^4 x^{\frac{1}{2}}\, dx;$

(ii) $\displaystyle\int_{-\pi}^{\pi} \sin(2x + \pi)\, dx.$ ***(5 marks)***

5 Using parametric differentiation, find $\dfrac{dy}{dx}$ in terms of t when $y = \ln(t^5)$ and $x = e^{3t}$.

(3 marks)

6 Let $y^2 = e^x + e^y$. Find $\dfrac{dy}{dx}$. ***(2 marks)***

7 (i) Solve $x^4 - 10x^2 + 9 = 0$. ***(2 marks)***

(ii) Find the stationary points of the curve

$$y = f(x) = x^4 - 10x^2 + 9$$

and determine their nature. ***(9 marks)***

(iii) Sketch the graph of $y = f(x) = x^4 - 10x^2 + 9$. ***(7 marks)***

(iv) Using your answers to (i) and (iii), or otherwise find

$$x^4 - 10x^2 + 9 < 0.$$

(2 marks)

8 (i) Express

$$\frac{3x^3 - 3x^2 + x + 1}{(x-1)^2(x^2+1)}$$

as partial fractions. Your answer should include a check. ***(14 marks)***

(ii) Find

$$\int \frac{3x^3 - 3x^2 + x + 1}{(x-1)^2(x^2+1)}\,dx.$$ ***(5 marks)***

9 Find $\displaystyle\int \frac{2x-1}{x+3}\,dx$. ***(4 marks)***

10 Find the area enclosed by the curve $y = x^2 - x - 2$ and the lines $x = 0$ and $x = 3$. ***(7 marks)***

11 Find the equations of the tangent and the normal to the curve $y = f(x) = \ln(x)$ at the point $P = (e^2, 2)$. ***(7 marks)***

12 Find the volume swept out when the area under $y = \sin x$ enclosed by $x = 0$ and $x = \pi$ is rotated about the x-axis through 2π radians. ***(5 marks)***

13 Find the equation of the curve $y = f(x)$ which passes through the point $\left(\frac{4\pi}{3}, 1\right)$ and has derivative

$$y' = f'(x) = \cos\left(\frac{1}{2}x + \frac{\pi}{3}\right).$$

(4 marks)

End of Question Paper