The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2013–2014**

Time Series **2 hours**

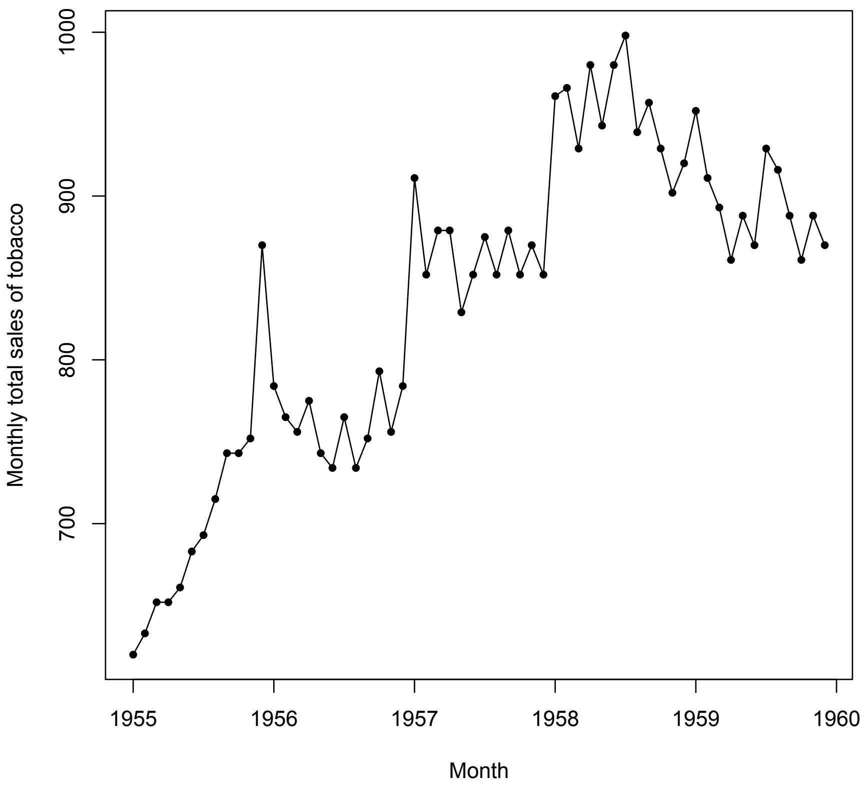
Marks will be awarded for your best ***three*** *answers.*

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (but no textbooks) plus a calculator that conforms to University regulations.
There are 99 marks available on the paper.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

Blank



1 (i) The plot above shows data consisting of monthly total sales (in some standard scale) of UK tobacco and related products in the period 1955 to 1959[1]. Briefly describe the features of the data. ***(4 marks)***

[1]Source: West, M. and Harrison, P.J. (1997) Bayesian Forecasting and Dynamic Models, Springer

1 (continued)

(ii) For a new time series z_t with length 20, the sample ACF and the sample PACF are tabulated below:

Lag	1	2	3	4	5
ACF	0.62	0.57	0.30	0.10	0.05

and

Lag	1	2	3	4	5
PACF	$a_1^{(1)}$	$a_2^{(2)}$	0.28	0.15	0.01

(a) Determine whether z_t is stationary or not and give a brief explanation. ***(2 marks)***

(b) Find the values of $a_1^{(1)}$ and $a_2^{(2)}$. ***(6 marks)***

(c) Test whether z_t is a white noise. ***(5 marks)***

(d) Test whether z_t is consistent with autoregressive models. ***(5 marks)***

(e) Test whether z_t is consistent with moving average models. ***(8 marks)***

(f) Based on your analysis above, suggest a time series model for z_t that is likely to perform well when fitted to the data. ***(3 marks)***

2 Consider the time series model

$$y_t = 6 - \frac{1}{4}y_{t-1} + \epsilon_t + \frac{1}{3}\epsilon_{t-1},$$

where ϵ_t is white noise with variance 2.

(i) Show that this model is causal and invertible. ***(4 marks)***

(ii) Find the mean of y_t. ***(5 marks)***

(iii) Find the variance of y_t. ***(10 marks)***

(iv) Find the autocorrelation function of y_t. ***(14 marks)***

3 Suppose that a model is set up for a seasonal time series y_t so that the transformed series x_t, defined by

$$x_t = (1 - B^4)^3 y_t,$$

where B is the backward shift operator, follows the AR(1) model

$$x_t = 0.7x_{t-1} + \epsilon_t,$$

where ϵ_t is white noise with variance 1.

(i) Write down the abbreviated form of the model of y_t: SARIMA$(p, d, q) \times (P, D, Q)_s$, i.e. identify p, d, q, P, D, Q and s. ***(2 marks)***

(ii) Show that

$$y_t = x_t + 3y_{t-4} - 3y_{t-8} + y_{t-12},$$

for $t > 12$. ***(8 marks)***

(iii) If the first 13 observations of y_t were

t	1	2	3	4	5	6	7	8	9	10	11	12	13
y_t	10	12	15	16	9	13	15	17	8	12	14	15	9

find the one-step, two-step and three-step forecast means of y_{14}, y_{15} and y_{16} respectively. ***(17 marks)***

(iv) If y_{14}, y_{15} and y_{16} were respectively 13, 16 and 18, then calculate the forecast error in each of the forecasts in part (iii) above and briefly comment on the forecast performance, based on these 3 predictions. ***(6 marks)***

4 Consider the time series model

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + z_t \gamma_t + \eta_t,$$

where α_1, α_2 are some parameters, z_t is a known time-varying covariate, γ_t is a time-varying regression parameter and η_t is a white noise with variance 10. The modeller postulates that $\gamma_t \approx \gamma_{t-1}$ (γ_t has a slow evolution), hence she suggests using the evolution model

$$\gamma_t = \gamma_{t-1} + \nu_t,$$

where ν_t is a white noise with variance 1. Assume that η_t and ν_t are independent for all t, each of them following a normal distribution.

(i) Define the state vector

$$\beta_t = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \gamma_t \end{bmatrix}.$$

Write the model of y_t in state-space form, i.e.

$$y_t = x_t^\top \beta_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2)$$
$$\beta_t = F\beta_{t-1} + \zeta_t, \quad \zeta_t \sim N(0, Z)$$

and determine the values of x_t, F, σ^2 and Z. ***(8 marks)***

(ii) If $z_3 = 2$, $y_1 = 5$, $y_2 = 7$, $y_3 = 4$, the posterior mean vector and the posterior covariance matrix of β_2 are

$$\hat{\beta}_{2|2} = \begin{bmatrix} 1 \\ 1/5 \\ 1 \end{bmatrix} \quad \text{and} \quad P_{2|2} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix},$$

then calculate the posterior mean vector and the posterior covariance matrix of β_3 at time $t = 3$. ***(19 marks)***

(iii) If $z_4 = 3$, find a 95% prediction interval for y_4. Based on this interval alone, comment on the forecast accuracy for the future observation y_4. ***(6 marks)***

End of Question Paper