The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2013-2014**

Mathematical Methods for Statistics **2 hours**

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.
*Candidates should attempt **ALL** questions.*
The paper will be marked out of 80 and the allocation of marks is shown in brackets.

1 Verify that

$$10^4 \sum_{n=0}^{\infty} \left(\tfrac{1}{101}\right)^n = \sum_{n=1}^{100} 2n.$$

(10 marks)

2 Write down the first four terms of the Taylor series expansion of $f(x) = e^{3x-1}$ about the point $x = 0$. **(10 marks)**

3 Find and classify *all* the critical points of the function

$$f(x, y) = x^3 + 2y^3 - 3x - 6y.$$

(10 marks)

4 Find:

(i)

$$\int x \sin(x^2)\mathrm{dx}$$

(ii)

$$\int_2^{10} \frac{1}{x^2 - x}\mathrm{dx}$$

(10 marks)

5 Let $f(x, y) = x + y - 1$ and $D \subset \mathbb{R}^2$ be the region bounded by the triangle with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$. Find

$$\iint_D f(x, y) dA.$$

(10 marks)

6 Use Gaussian elimination to solve the following system of equations:

$$\begin{aligned} 3x + 2y + 3z - 2w &= 1; \\ x + y + z &= 3; \\ x + 2y + z - w &= 2. \end{aligned}$$

(10 marks)

7 Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

Show that $\{\mathbf{v}, A\mathbf{v}, A^2\mathbf{v}\}$ is a basis for $\mathbb{R}^3$. **(10 marks)**

8 Let a, b be real numbers. Find

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix}^{2014}$$

Hint: It is easy to take powers of diagonal matrices. **(10 marks)**

End of Question Paper