



The
University
Of
Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS

June 2014

Probability and Probability Distributions

2 hours

RESTRICTED OPEN BOOK EXAMINATION.

Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.

*Candidates should attempt **ALL** questions.*

The maximum marks for the various parts of the questions are indicated.

The paper will be marked out of 80.

**Please leave this exam paper on your desk
Do not remove it from the hall**

Registration number from U-Card (9 digits)
to be completed by student

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- 1** Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k/(x+1) & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the value of k . *(4 marks)*
- (b) Find the distribution function of X . *(5 marks)*
- (c) Find $E(X)$. *(3 marks)*
- (d) Let $Y = 1/X$. Find the probability density function of Y . *(5 marks)*
- 2** A fair coin is tossed three times. Assume the tosses are independent of each other.

- (a) Describe an appropriate sample space to use for this experiment. *(3 marks)*
- (b) Let X be the random variable which counts the number of heads observed in the experiment.
- (i) For each $i \in \{0, 1, 2, 3\}$, give the elements of the sample space which are in the event $\{X = i\}$ and the probability that $X = i$. *(8 marks)*
- (ii) Find the mean and variance of the random variable X . *(7 marks)*
- (c) Let E be the event that there are more heads than tails, and let F be the event that the number of heads is odd. Are these events independent? Give a reason for your answer. *(3 marks)*

- 3** Let R be the region defined by $R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$, and let X and Y be random variables with joint probability density function given by

$$f_{X,Y}(x, y) = \begin{cases} \frac{3}{4}(x-y)^2 & (x, y) \in R \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find $P(X \leq 1, Y \leq 1/2)$. *(4 marks)*
- (b) Find the marginal probability density functions of X and Y . *(5 marks)*
- (c) Are X and Y independent? Give a reason for your answer. *(3 marks)*
- (d) Find $E(XY)$. *(4 marks)*

- 4 (a) State which standard distribution might serve as a model for each of the following.
- (i) The number of meteorites above a certain size hitting a particular area of the Earth in a year. *(2 marks)*
 - (ii) The number of items in a batch of fixed size which are faulty. *(2 marks)*
 - (iii) The length of time until the next meteorite above a certain size hits a particular area of the Earth. *(2 marks)*
- (b) In (a) parts (i) and (ii), under what circumstances might a Normal approximation to the distribution you suggested be appropriate? *(4 marks)*
- 5 A patient takes a test to see whether they have a disease. If the patient has the disease, then the test will be positive with probability p and negative with probability $1 - p$, while if the patient does not have the disease the test will be positive with probability q and negative with probability $1 - q$. According to the patient's doctor, the prior probability that they have the disease is d .
- (a) Find the posterior probability that the patient has the disease, in terms of p , q and d , if
- (i) the test result is positive;
 - (ii) the test result is negative. *(8 marks)*
- (b) It is considered that it is worthwhile to treat the patient for the disease if the posterior probability of them having the disease is at least 0.4.
- (i) If $p = 0.9$ and $q = 0.1$, for what values of d will a positive test indicate that it will be worthwhile for the patient to be treated for the disease? *(4 marks)*
 - (ii) If $p = 0.9$ and $q = 0.1$, for what values of d will it be worthwhile to treat the patient for the disease even if the test result is negative? *(4 marks)*

End of Question Paper