The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2014-2015**

CORE FOUNDATION MATHEMATICS **3 hours**

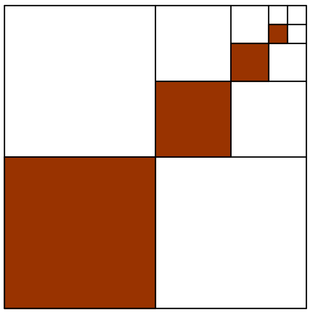
*Attempt **all** questions. The allocation of marks is shown in brackets. Total marks: 100.*

1 (i) Find

(a) $\displaystyle\sum_{r=1}^{100}(2r+1)\,,$

(b) $\displaystyle\sum_{r=1}^{\infty}\left(\frac{1}{10}\right)^{r}\,.$ ***(6 marks)***

(ii) In the diagram, $\frac{1}{4}$ of the large square is initially shaded. Then $\frac{1}{4}$ of the square in the top right hand corner is shaded. This pattern of shading is continued infinitely.



Show that the shaded areas form an infinite geometric series. Hence find the fraction of the large square that is eventually shaded. ***(8 marks)***

(iii) Find the first four terms when $\left(1+\frac{1}{2}x\right)^{10}$ is expanded in ascending powers of x. ***(6 marks)***

2 (i) Let $f(x) = 3\cos x - 6\sin x$.

(a) Express $f(x) = 3\cos x - 6\sin x$ in the form $R\cos(x+\alpha)$, $0 < \alpha < \dfrac{\pi}{2}$.

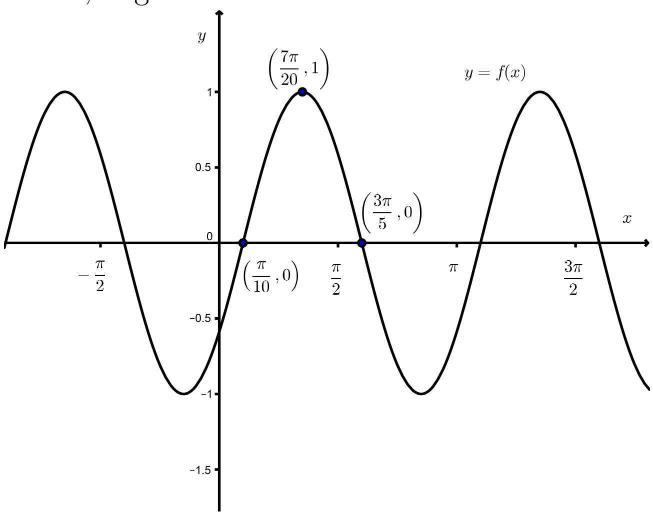
(b) Determine the maximum value of $f(x)$ and the values of x for which this maximum value occurs.

(c) Determine the minimum value of $g(x) = f(x) + 3$ and the values of x for which this minimum value occurs.

(6 marks)

(ii) Prove that $\tan x + \cot x = \operatorname{cosec} x . \sec x$, where $0 < x < \dfrac{\pi}{2}$. ***(4 marks)***

(iii) The graph of $y = \sin(ax - b)$, where a and b are positive constants and $b < \pi$, is given below:



It is given that the points with coordinates $\left(\dfrac{\pi}{10}, 0\right)$, $\left(\dfrac{7\pi}{20}, 1\right)$ and $\left(\dfrac{3\pi}{5}, 0\right)$ lie on the graph of $y = \sin(ax - b)$. Find a and b. ***(6 marks)***

3 (i) A circle C has equation $(x+5)^2+(y-12)^2=196$.

(a) Find the coordinates of the centre and the radius of C.

(b) Show that the point P with coordinates $\left(\frac{5}{13}, -\frac{12}{13}\right)$ lies on C.

(6 marks)

(ii) Find the Cartesian equation of the curve described parametrically as

$$x = 2\cot(t)\,; \qquad y = \operatorname{cosec}(\mathrm{t}) \qquad \text{for t} \neq 0 \text{ and } -\frac{\pi}{2} < \mathrm{t} < \frac{\pi}{2}.$$

(4 marks)

(iii) Given that $t=\ln x$ express the following in terms of t:

$$\text{(a)}\ \ln\left(x^2\right)\,, \qquad \text{(b)}\ \frac{\ln(x^{\frac{1}{2}})}{\ln\left(e^2\right)}.$$

(4 marks)

4 (i) Find the derivative of $f(x)=x^2-3$ by using *first principles.*

(ii) Differentiate $y=x^7-\dfrac{2}{x^{\frac{1}{3}}}+\sqrt[5]{x}-9$.

(iii) Differentiate $y=\sin(e^x)$.

(iv) Differentiate $y=(\ln x)\sin^{-1}x$.

(v) Differentiate $y=\dfrac{\tan x}{x}$.

(vi) Find $\dfrac{dy}{dx}$ in terms of t when $y=\dfrac{t}{t^2+1}$ and $x=t^2+1$.

(vii) Find $\dfrac{dy}{dx}$ if $y=x^{\cos x}$. ***(11 marks)***

5 (i) Find $\displaystyle\int\left(3x^2-\frac{4}{x^{\frac{3}{7}}}-\frac{1}{x}+e\right)dx$.

(ii) Find $\displaystyle\int 5xe^{(x^2)}\,dx$.

(iii) Find $\displaystyle\int x\cos x\,dx$.

(iv) Find $\displaystyle\int_0^{\pi}\cos\left(4x-\frac{\pi}{2}\right)dx$. ***(7 marks)***

6 Let $y = \ln\left(\dfrac{e^x x^6 \sin^4 x}{e^5(\tan^{-1} x)x^{\frac{3}{4}}}\right)$.

(i) Simplify $y = \ln\left(\dfrac{e^x x^6 \sin^4 x}{e^5(\tan^{-1} x)x^{\frac{3}{4}}}\right)$ as much as possible using the laws of logarithms.

(ii) Use your answer to (i) to find $\dfrac{dy}{dx}$. ***(5 marks)***

7 Let $y = f(x) = x^3(x^2 - 1)^2$.

(i) Find the stationary points of $y = f(x) = x^3(x^2 - 1)^2$ and determine their nature. ***(12 marks)***

(ii) Sketch the graph of $y = f(x) = x^3(x^2 - 1)^2$. ***(5 marks)***

(iii) Find the *area* between $x = -1$, $x = 1$, the curve $y = x^3(x^2 - 1)^2$ and the x-axis. ***(3 marks)***

8 Let $p(x) = \dfrac{2x^3 + 2x^2 + x + 1}{x^2(x^2 + 1)}$.

(i) Express $p(x)$ in partial fractions. Your solution should include a check.

(ii) Using your answer to part (i) or otherwise, find

$$\int \frac{2x^3 + 2x^2 + x + 1}{x^2(x^2 + 1)}\, dx\,.$$

(7 marks)

End of Question Paper