The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2014–2015**

Further Foundation Mathematics **2 hours**

Answer ***all questions****.*
You should justify your answers carefully unless the question states otherwise.

1 (i) Suppose $r = 5 - 7i$ and $s = 1 + 2i$.

(a) Evaluate $r + s$, $r - s$, rs and r/s exactly. ***(6 marks)***

(b) Put r in polar form, giving the modulus and argument as decimals accurate to 4 significant figures (using radians for the argument). ***(3 marks)***

(c) Calculate r^5, giving the answer in polar form as above. ***(3 marks)***

(ii) (a) Describe (in words) the locus C of points in the plane whose coordinates (x, y) satisfy

$$x^2 - 4x + y^2 + 6y = 87.$$

(4 marks)

(b) Show that the line with equation $y = -\dfrac{3}{4}x + 11$ is a tangent line to C, and find the point of tangency. ***(4 marks)***

2 (i) In this question, let $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$.

(a) What is $\mathbf{u} + \mathbf{v}$? What is $\mathbf{u} - \mathbf{v}$? What is $\mathbf{u} \cdot \mathbf{v}$? What is $\mathbf{u} \times \mathbf{v}$? ***(6 marks)***

(b) What is the angle between $\mathbf{u}$ and $\mathbf{v}$ (in radians to four significant figures)? ***(3 marks)***

(c) Find, to four decimal places, the values of λ that would make the angle between $\mathbf{u}$ and $\mathbf{u} + \lambda\mathbf{v}$ equal to $\pi/3$. ***(5 marks)***

(ii) Let ℓ_1 be the line with equation

$$\mathbf{x} = (\mathbf{i} - 6\mathbf{j} + 8\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

and ℓ_2 be the line with equation

$$\mathbf{x} = (-4\mathbf{i} + \mathbf{j} + \mathbf{k}) + \mu(-3\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Do these lines meet? If so, find the position vector of the point where they do meet. ***(6 marks)***

3 (i) Sketch a graph of the function

$$y = \frac{1 - x}{2 - x},$$

labelling clearly the coordinates of the points where it meets the axes.

Where is it discontinuous? What are the asymptotes? ***(7 marks)***

(ii) (a) Write down the binomial expansion for $(1+4x)^{-1/2}$ up to (and including) the x^4 term. ***(5 marks)***

(b) Show that the Maclaurin series for $\cos(x)$ up to (and including) the x^4 term is given by

$$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \cdots .$$

(4 marks)

(c) Multiplying the above two together, write down the Maclaurin series for $\dfrac{\cos(x)}{\sqrt{1+4x}}$ up to (and including) the x^4 term. ***(4 marks)***

4 (i) (a) By calculating $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, show that, for any constant a, the functions $y = \sin(x + a) + 1$ satisfy the differential equation

$$2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y^2 = 2.$$

(6 marks)

(b) Hence find a solution to that equation which has $y = \frac{1}{2}$ when $x = 0$.

(4 marks)

(ii) (a) Find the general solution to the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$

(6 marks)

(b) Hence find a solution to the above equation which has $y = 3$ when $x = 0$ and $y = e^2 + 2e$ when $x = 1$. ***(4 marks)***

End of Question Paper