Data provided: Formula sheet

The University Of Sheffield.

MAS153/MAS159

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2014–2015**

Mathematics (Materials)
Mathematics For Chemists **3 hours**

All questions are compulsory. The marks awarded to each question or section of question are shown in italics.

1 Obtain the general solution of the equation

$$\sin 2\theta = 2\cos\theta + \sin\theta - 1.$$ ***(7 marks)***

2 Factorise the polynomial

$$x^3 + x^2 - x - 1.$$ ***(2 marks)***

3 Find the equations of the tangent and the normal to the curve $y^2 = x^3 + x + 1$ at the point (0,1). ***(7 marks)***

4 The equation

$$(x + y)^2 = 4(xy + 1)$$

reduces to two equations of parallel straight lines. Write down the equations of those straight lines.
[Hint: Expand the brackets and simplify the equation.] ***(6 marks)***

5 Determine whether the point $(9, 14)$ lies on the line given by the parametric equations:

$$x = 1 + 2t \quad \text{and} \quad y = 2 + 3t.$$ ***(2 marks)***

6 Solve the following equation for real x:

$$4^x + 2^x - 2 = 0.$$ ***(6 marks)***

7 Differentiate, with respect to t, the functions

(a) $\ln(2t^2+2)$, *(2 marks)*

(b) $\sin^3 t$. *(2 marks)*

8 Find the following integral

$$\int \frac{4x+3}{x^2+1}\,dx.$$ *(4 marks)*

9 Evaluate $\int_{-1}^{2} |x|\,dx$ where

$$|x| = \begin{cases} -x & x<0, \\ x & x>0. \end{cases}$$

Note that the function $|x|$ is not differentiable at $x=0$. *(5 marks)*

10 Find the domains of the following functions:

(a) $\ln(1-x)$, (b) $\frac{1}{x^2}$. *(4 marks)*

11 Find the arithmetic and geometric mean of 1, 2, 32. *(3 marks)*

12 (a) Showing your working clearly, find the coefficient of x^3 in the expansion of $(1+x)^{27}$. *(2 marks)*

(b) Use the binomial theorem to evaluate

$$\lim_{x\to\infty}\left[\sqrt{x^2+8x+3}-x-3\right].$$ *(3 marks)*

13 Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by

$$\mathbf{a} = (-1,2,1), \quad \mathbf{b} = (1,-2,3), \quad \mathbf{c} = (2,0,1).$$

(a) Verify that

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}).$$ *(6 marks)*

(b) Verify that

$$\mathbf{b}\times(\mathbf{a}\times\mathbf{c}) = (\mathbf{b}\cdot\mathbf{c})\mathbf{a}-(\mathbf{b}\cdot\mathbf{a})\mathbf{c}.$$ *(6 marks)*

14 Prove, from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, the identity

$$\cosh x\,\cosh y + \sinh x\,\sinh y = \cosh(x+y).$$ *(3 marks)*

15 Evaluate

$$\int_0^1 \frac{3x^2+2x-7}{(x-2)(x^2-x-2)}\,dx.$$ *(9 marks)*

16 Find the first 3 terms of the Maclaurin series for $\dfrac{1}{2-e^x}$. *(5 marks)*

17 Express the complex number

$$z = 24 + 7i$$

in polar form. *(2 marks)*

Find the four values of $z^{1/4}$ in exponential form, and plot them on an Argand diagram. *(4 marks)*

18 A set of linear equations can be written as

$$\begin{pmatrix} -\lambda & 0 & 2 \\ 3 & 1 & \lambda-4 \\ 0 & -1 & -1 \end{pmatrix} X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

where

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

Find the values of λ for which non-zero solutions X can exist. *(4 marks)*

For each of these values of λ, find the corresponding solution X. *(6 marks)*

End of Question Paper