Data provided: Formula sheet

The University Of Sheffield.

MAS157

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2014–2015**

Mathematics For Chemists **2 hours**

All questions are compulsory. The marks awarded to each question or section of question are shown in italics.

1 (a) Showing your working clearly, find the coefficient of x^3 in the expansion of $(1+x)^{27}$. ***(3 marks)***

(b) Use the binomial theorem to evaluate

$$\lim_{x\to\infty}\left[\sqrt{x^2+8x+3}-x-3\right].$$ ***(5 marks)***

(c) Use the first three terms of the binomial theorem to estimate $\sqrt[4]{0.97}$, giving your answer to 9 decimal places. ***(6 marks)***

2 Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by

$$\mathbf{a}=(-1,2,1), \quad \mathbf{b}=(1,-2,3), \quad \mathbf{c}=(2,0,1).$$

(a) Verify that

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}).$$ ***(8 marks)***

(b) Verify that

$$\mathbf{b}\times(\mathbf{a}\times\mathbf{c})=(\mathbf{b}\cdot\mathbf{c})\mathbf{a}-(\mathbf{b}\cdot\mathbf{a})\mathbf{c}.$$ ***(10 marks)***

3 (a) Prove, from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, the identity

$$\cosh x\,\cosh y + \sinh x\,\sinh y = \cosh(x+y).$$ *(4 marks)*

(b) Find $\dfrac{dy}{dx}$ if

$$y = \cosh^{-1} x.$$ *(7 marks)*

4 Evaluate

$$\int_0^1 \frac{3x^2+2x-7}{(x-2)(x^2-x-2)}\,dx.$$ *(14 marks)*

5 Find the first 3 terms of the Maclaurin series for $\dfrac{1}{2-e^x}$. *(8 marks)*

6 (a) Express the complex number

$$z = 24 + 7i$$

in polar form. *(3 marks)*

Find the four values of $z^{1/4}$ in exponential form, and plot them on an Argand diagram. *(6 marks)*

(b) If z is a complex number, find all the possible solutions of

$$|z+1| = 2z.$$ *(12 marks)*

7 A set of linear equations can be written as

$$\begin{pmatrix} -\lambda & 0 & 2 \\ 3 & 1 & \lambda-4 \\ 0 & -1 & -1 \end{pmatrix} X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

where

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

Find the values of λ for which non-zero solutions X can exist. *(6 marks)*

For each of these values of λ, find the corresponding solution X. *(8 marks)*

End of Question Paper