The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2014–2015**

Time Series **2 hours**

Marks will be awarded for your best ***three*** *answers.*

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (but no textbooks) plus a calculator that conforms to University regulations.
There are 60 marks available on the paper.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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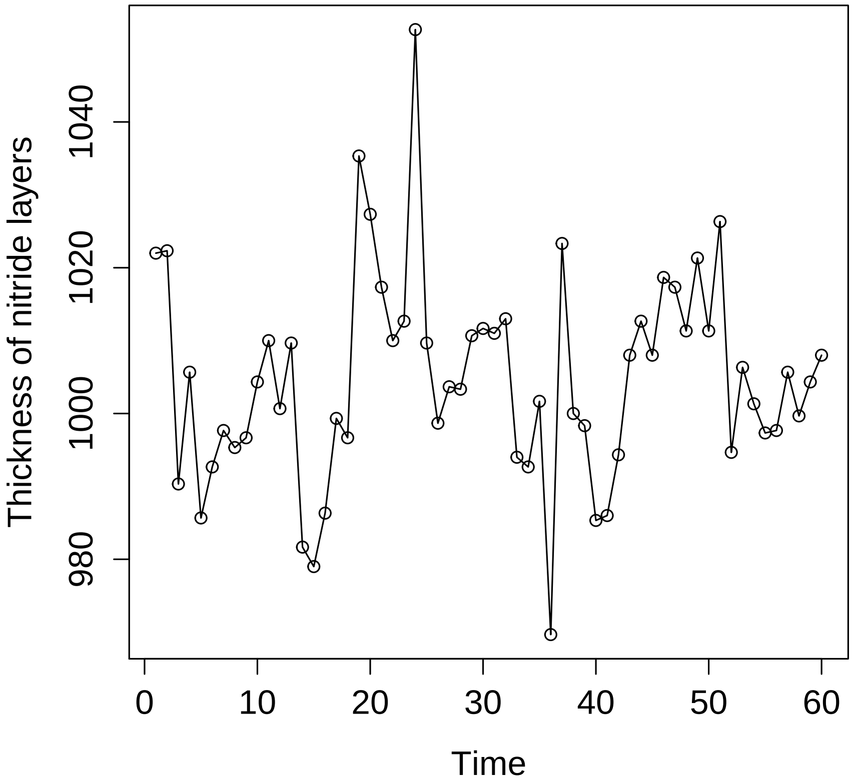

1 (i) The plot above shows time series data consisting of 60 observations of the thickness of nitride layers (unknown units); the data was part of a larger experiment on the manufacturing of a microelectronic device [1].

(a) Describe the data by commenting on their structure, their variation and dynamics. ***(2 marks)***

(b) Based on your answer in (a) or otherwise, suggest suitable time series model(s) that may be appropriate for this data. ***(2 marks)***

[1] Source: Triantafyllopoulos, K., Godolphin, J.D. and Godolphin, E.J., 2005, Process improvement in the microelectronic industry by state space modelling, Quality and Reliability Engineering International, 21, 465-475

 Question 1 continued on next page

1 (continued)

(ii) A model is to be fitted to a time series of length 100. Values of the sample autocorrelation function (ACF) and sample partial ACF (PACF) are tabulated below.

Lag (h)	1	2	3	4	5
ACF (r_h)	0.6	0.4	0.1	0.05	0.01
PACF ($\hat{a}_h^{(h)}$)	$\star$	$\star\star$	0.02	0.01	-0.02

(a) Find the omitted values ($\star$ and $\star\star$). ***(4 marks)***

(b) Check whether the time series is stationary. ***(1 mark)***

(c) Test whether the time series is consistent with white noise. ***(2 marks)***

(d) Test whether the time series is consistent with moving average models. ***(4 marks)***

(e) Test whether the time series is consistent with autoregressive models. ***(3 marks)***

(f) Based on your answer in (c)-(e) above, suggest a time series model that may be suitable to model the data. ***(2 marks)***

2 Consider the time series model

$$y_t = -\frac{1}{3}y_{t-1} + \epsilon_t - \frac{1}{2}\epsilon_{t-1} + \frac{1}{4}\epsilon_{t-2},$$

where ϵ_t is white noise with variance 1.

(i) Write the above model using the backward shift operator B. ***(2 marks)***

(ii) Show that this model is causal and invertible. ***(3 marks)***

(iii) Find the mean and the variance of y_t. ***(6 marks)***

(iv) Find the autocorrelation function of y_t. ***(9 marks)***

3 (i) In the context of maximum likelihood estimation of ARMA models describe briefly what is meant by conditional least squares estimation. ***(2 marks)***

(ii) Consider that y_t is generated by an autoregressive model of order 2 (AR(2))

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \epsilon_t,$$

where α_1, α_2 are the AR coefficients and ϵ_t is white noise with variance σ^2.

(a) Write down the conditional likelihood and the conditional log-likelihood functions of the parameters α_1, α_2 and σ^2, based on a collection of observations $y_{1:n} = (y_1, y_2, \dots, y_n)$. ***(4 marks)***

(b) Using conditional least squares, show that the maximum likelihood estimates of α_1, α_2 and σ^2 are

$$\hat{\alpha}_1 = \frac{\sum_{t=3}^n y_{t-2}^2 \sum_{t=3}^n y_t y_{t-1} - \sum_{t=3}^n y_{t-1} y_{t-2} \sum_{t=3}^n y_t y_{t-2}}{\sum_{t=3}^n y_{t-1}^2 \sum_{t=3}^n y_{t-2}^2 - \left(\sum_{t=3}^n y_{t-1} y_{t-2}\right)^2}$$

$$\hat{\alpha}_2 = \frac{\sum_{t=3}^n y_{t-1}^2 \sum_{t=3}^n y_t y_{t-2} - \sum_{t=3}^n y_t y_{t-1} \sum_{t=3}^n y_{t-1} y_{t-2}}{\sum_{t=3}^n y_{t-1}^2 \sum_{t=3}^n y_{t-2}^2 - \left(\sum_{t=3}^n y_{t-1} y_{t-2}\right)^2}$$

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum_{t=3}^n \left(y_t - \hat{\alpha}_1 y_{t-1} - \hat{\alpha}_2 y_{t-2}\right)^2.$$

(14 marks)

4 A company trades 10 products, with the ith product projected to give a return r_{it} at time t, for $i = 1, 2, \ldots, 10$. The company believes that each of these returns r_{it} follows an autoregressive process

$$r_{it} = 0.9 r_{i,t-1} + \zeta_{it},$$

where ζ_{it} is a white noise with variance 1, $\zeta_{it} \sim N(0, 1)$, and ζ_{it} is independent of ζ_{jt}, for any $i \neq j$.

Due to a data recording error r_{it} is not available. However, the aggregate return can be observed subject to additive noise, according to the model

$$y_t = \sum_{i=1}^{10} r_{it} + \epsilon_t,$$

where ϵ_t is a white noise with variance 1, $\epsilon_t \sim N(0, 1)$, and it is assumed that ϵ_t is independent of ζ_{it}, for any t and for any i.

(i) Define the state

$$\beta_t = \sum_{i=1}^{10} r_{it}.$$

Show that y_t follows a state space model

$$y_t = x\beta_t + \epsilon_t$$
$$\beta_t = F\beta_{t-1} + \zeta_t$$

and determine x, F, ζ_t and the variance of ζ_t. ***(4 marks)***

(ii) A prior distribution for β_0 is set as

$$\beta_0 \sim N(0, 100).$$

If the first observation is $y_1 = 2$, perform the Kalman filter iteration for $t = 1$ and obtain the posterior distribution of

$$\beta_1 \mid \{y_1 = 2\}.$$

(8 marks)

(iii) Using the result in (ii) obtain a 95% predictive interval for y_2. ***(4 marks)***

(iv) Describe briefly what is the likely effect on the posterior distribution of β_t (for large t), if the prior distribution of β_0 changes from (a) $\beta_0 \sim N(0, 1)$ to (b) $\beta_0 \sim N(0, 1000)$. ***(4 marks)***

End of Question Paper