The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2015-2016**

CORE FOUNDATION MATHEMATICS **3 hours**

*Attempt **all** questions. The allocation of marks is shown in brackets. Total marks: 100.*

1 (i) Let $f(x) = 4\cos x - 5\sin x$.

(a) Write $f(x)$ in the form

$$f(x) = R\cos(x+\alpha),\ 0 < \alpha < \frac{\pi}{2},$$

expressing α to 2 d.p.

(b) Determine the maximum value of $f(x)$ and the values of x for which this maximum value occurs.

(c) Determine the minimum value of $F(x) = f(x) - 2$ and the values of x for which this minimum value occurs. ***(8 marks)***

(ii) Prove that $\tan x = \dfrac{2}{\sin 2x} - \dfrac{1}{\tan x}$, where $0 < x < \dfrac{\pi}{2}$. ***(4 marks)***

2 (i) A circle C has equation $(x-4)^2 + (y+1)^2 = 25$.

(a) Find the coordinates of the centre of the circle.

(b) Find the radius of the circle.

(c) Show that the point P with coordinates $(0, 2)$ lies on C.

(4 marks)

(ii) Find the cartesian equation of the curve described parametrically as

$$x = 2\tan t, \qquad y = \sec t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(4 marks)

3 (i) Find

(a) $\displaystyle\sum_{q=1}^{\infty}\left(-\frac{1}{6}\right)^{q}$,

(b) $\displaystyle\sum_{t=1}^{50}(4t+1)$. ***(6 marks)***

(ii) Showing all your workings, (*including* workings for the binomial coefficients), find the first three terms when $\left(1+\frac{y}{4}\right)^{12}$ is expanded in ascending powers of y. ***(6 marks)***

(iii) A frog wants to move from point A to point B. Each time the frog jumps, it lands halfway between its starting point and the point B.

Let d be the distance between the point A and the point B.

(a) For *each* of the frog's first three jumps, write down the *individual* distance covered as a fraction of d.

(b) Explain why these fractions are the first three terms of a geometric sequence.

(c) What would be the distance from A covered by the frog after n jumps?

(d) What would be the distance from A covered by the frog as $n \to \infty$? Will the frog ever reach the point B? Briefly explain your answer. ***(7 marks)***

4 (i) A doctor gives a patient 5mg of medicine. The medicine leaves the patient's bloodstream at a rate of 10% per hour.

Determine how long it takes (to the nearest hour) for the amount of medicine in the patient's bloodstream to fall below 1mg. ***(6 marks)***

(ii) The relative intensity, I, of an earthquake can be given as a magnitude, M, on a base-10 logarithmic scale called the Richter scale.

(a) Write down an equation that shows how this magnitude, M relates to the relative intensity, I.

(b) Two earthquakes X and Y have magnitudes $M_X = 2$ and $M_Y = 5$. Using your equation from part (i) (a), determine how many times more intense earthquake Y is than earthquake X. ***(5 marks)***

5 (i) Find the derivative of $f(x) = 4x - x^2$ by using *first principles*.

(ii) Differentiate $y = x^3 + \dfrac{5}{x^{\frac{2}{7}}} - \sqrt[3]{x} + 17$.

(iii) Differentiate $y = e^{\tan x}$.

(iv) Differentiate $y = (\ln x)(\tan^{-1} x)$.

(v) Differentiate $y = \dfrac{\cos x}{x^2 + 1}$.

(vi) Find $\dfrac{dy}{dx}$ in terms of t when $y = \dfrac{t^3 + 1}{t^7 - 1}$ and $x = \dfrac{t^2 + 1}{t^3}$.

(vii) Find $\dfrac{dy}{dx}$ if $y = 2^{e^{\sin x}}$. ***(13 marks)***

6 (i) Find $\displaystyle\int \left(4x^3 + \frac{3}{x^{\frac{5}{9}}} + \frac{1}{x} + \pi\right) dx$.

(ii) Find $\displaystyle\int 3x \cos(x^2 + 2)\, dx$.

(iii) Find $\displaystyle\int xe^x\, dx$.

(iv) Find $\displaystyle\int_0^1 \frac{1}{(2x)^2 + 1}\, dx$. ***(7 marks)***

7 Let $y = f(x) = \dfrac{1}{x^2 + 1} - \dfrac{1}{x^2 + 2}$.

(i) Find the stationary points of $y = f(x) = \dfrac{1}{x^2 + 1} - \dfrac{1}{x^2 + 2}$ and determine their nature. ***(5 marks)***

(ii) Sketch the graph of $y = f(x) = \dfrac{1}{x^2 + 1} - \dfrac{1}{x^2 + 2}$. ***(6 marks)***

8 (i) Express $p(x) = \dfrac{x - 3}{(2x - 1)(x^2 - 4)}$ in partial fractions. ***(6 marks)***

(ii) Find

$$\int \frac{x - 3}{(2x - 1)(x^2 - 4)}\, dx$$

using your answer to (i) or otherwise. ***(2 marks)***

9 Find the area enclosed by the lines $y = \ln(x+1)$, $y = x^2 - 2x + \ln 3$, $x = 1$ and $x = 2$.
Give your answer as an approximation to two decimal points. ***(8 marks)***

10 Find the equation of the function $y = f(x)$ which has derivative $y' = \cos\left(2x - \frac{\pi}{2}\right)$ and has $y = 0$ when $x = 0$. ***(3 marks)***

End of Question Paper