The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2015–2016**

Further Foundation Mathematics **2 hours**

Answer ***all questions****.*
You should justify your answers carefully unless the question states otherwise.

1 (i) (a) Calculate $(1+i)(1-2i)$. ***(2 marks)***

(b) Calculate $(63+16i)/(3-4i)$. ***(2 marks)***

(ii) Solve the equation $3z - i = 6 - 2iz$. ***(3 marks)***

(iii) Let $z = \frac{1}{2}(i - \sqrt{3})$.

(a) Show that $z^3 = i$ directly, and deduce the value of z^{12}. ***(3 marks)***

(b) Put z into polar form, and hence calculate z^{12} another way. ***(3 marks)***

(iv) (a) Give a geometric description of the locus C of points satisfying

$$x^2 - 8x + y^2 + 10y + 16 = 0.$$

(3 marks)

(b) Which of the following lines are tangent lines to C?

(α) The x-axis;

(β) The y-axis;

(γ) The line $y = x + 3$.

(4 marks)

2 In this question, let $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{w} = 2\mathbf{j} - 2\mathbf{k}$.

(i) Find $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{w}$, $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{u} \times \mathbf{v}$ and $\mathbf{w} \times \mathbf{w}$. ***(5 marks)***

(ii) (a) What is the angle between $\mathbf{u}$ and $\mathbf{w}$? Give your answer in radians to three significant figures. ***(4 marks)***

(b) Find a value of λ such that the vector $\mathbf{u} + \lambda\mathbf{v}$ is perpendicular to $\mathbf{w}$. ***(3 marks)***

(c) Find a value of λ such that $\mathbf{u} + \lambda\mathbf{v}$ makes the same angle with $\mathbf{i}$ as with $\mathbf{j}$. ***(4 marks)***

(iii) Where does the line with equation $\mathbf{x} = \mathbf{u} + \lambda\mathbf{v}$ meet the plane with equation $\mathbf{x} = \mathbf{i} + \mu\mathbf{j} + \nu\mathbf{k}$? ***(4 marks)***

3 (i) Evaluate the sum

$$\sum_{n=1}^{100} \frac{n}{5}.$$

(3 marks)

(ii) (a) By considering the graphs of $y = 1 + x$ and $y = -e^{-x}$ first, sketch a graph of $y = 1 + x - e^{-x}$, labelling where it meets the axes. There is no need to describe the stationary points. ***(3 marks)***

(b) Sketch a graph of $y = 1/(|x| - 1)$, labelling where it meets the axes, and showing any points of discontinuity. There is no need to describe the stationary points. ***(3 marks)***

(iii) (a) Write down the Maclaurin series for $e^{3x} - 1$ up to and including the x^3 term. ***(4 marks)***

(b) Use the binomial theorem to write down the power series for $\sqrt[3]{1+y}$ up to and including the y^3 term. ***(4 marks)***

(c) Taking $y = e^{3x} - 1$, substitute the power series for $e^{3x} - 1$ into that for $\sqrt[3]{1+y}$ to get the power series for $\sqrt[3]{1 + e^{3x} - 1}$. Explain what you find. ***(3 marks)***

4 (i) (a) Find the general solution to the differential equation

$$\frac{d^2y}{dx^2} - 11\frac{dy}{dx} + 24y = 0.$$

(6 marks)

(b) Hence find the solution to that equation with $y = 1$ at $x = 0$ and $y = e^3$ at $x = 1$. ***(4 marks)***

(ii) Use the method of separation of variables to give the general solution to the differential equation

$$y\frac{dy}{dx} = \sin x.$$

(5 marks)

(iii) By calculating the first and second derivatives and comparing both sides, show that $y = e^{x^2}$ gives a solution to the differential equation

$$y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 2y^2.$$

(5 marks)

End of Question Paper