Data provided: Formula sheet

MAS157

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2015–2016**

Mathematics For Chemists **2 hours**

All questions are compulsory. The marks awarded to each question or section of question are shown in italics.

1 Let $y = e^{x^2+\cos x}$. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. ***(7 marks)***

2 Verify the following identity

$$\frac{\cosh^4 x - \frac{1}{2}\sinh^2(2x) + \sinh^4 x}{1 - \tanh^2 x} = \cosh^2 x.$$ ***(9 marks)***

3 (a) Showing your working clearly, find the coefficient of x^3 in the expansion of $(1+x)^{12}$. ***(3 marks)***

(b) Use the binomial theorem to evaluate

$$\lim_{x\to\infty}(\sqrt{x^2+6x-4} - x - 2).$$ ***(5 marks)***

4 (a) Show that the vectors $\mathbf{u} = (3, 0, 3)$ and $\mathbf{v} = (2, 12, -2)$ are perpendicular. ***(3 marks)***

(b) A plane passes through the points $\mathbf{a} = (0, 0, 1)$, $\mathbf{b} = (1, 1, -1)$ and $\mathbf{c} = (2, -2, 1)$. Find the Cartesian equation of the plane. ***(11 marks)***

5 Prove, from the definitions of $\sinh x$ and $\cosh x$, the identity

$$2 \sinh x \cosh x = \sinh 2x.$$ *(5 marks)*

6 Evaluate

$$\int \coth x \, dx.$$ *(8 marks)*

7 Evaluate

$$\int \frac{2x+1}{\sqrt{x^2+4}} \, dx.$$ *(13 marks)*

8 Find the Maclaurin series for $f(x) = e^{x^2+1}$, as far as the term in x^3. *(10 marks)*

9 Complex numbers z_1 and z_2 are defined by

$$z_1 = 1 + i, \qquad z_2 = 2 - i.$$

Find, in the form $a + bi$ where a and b are real,

(a) z_1^3, *(4 marks)*

(b) $\dfrac{z_2}{2z_1 + z_2}$. *(5 marks)*

10 Solve the system of linear equations

$$\begin{cases} x + 2y + z = 3, \\ x + 3y + 2z = 4, \\ 2x + 5y + 3z = 7. \end{cases}$$ *(9 marks)*

11 Find the volume V of the parallelepiped determined by the vectors $\mathbf{a} = (1, 1, 1)$, $\mathbf{b} = (1, 1, 0)$ and $\mathbf{c} = (2, 1, 1)$. *(8 marks)*

End of Question Paper