Data provided: Formula sheet

MAS153/MAS159

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2015–2016**

Mathematics (Materials)
Mathematics For Chemists **3 hours**

All questions are compulsory. The marks awarded to each question or section of question are shown in italics.

1 Find the constants r and α, with $r > 0$ and $-\pi < \alpha \leq 0$, such that

$$\cos x - \sqrt{3}\sin x = r\cos(x - \alpha).$$ ***(4 marks)***

Hence find all solutions x of the equation

$$\cos x - \sqrt{3}\sin x = \sqrt{2}.$$ ***(4 marks)***

2 Solve the equation

$$2\ln x - \ln(x-3) = \ln(x-6).$$ ***(6 marks)***

3 Identify the number d such that

$$\frac{4^{9/2}3^{11}}{6(3^5)^2 8^{5/3}} = 2^d.$$ ***(4 marks)***

4 Simplify

$$\frac{(27x^6y^9z^6)^{\frac{1}{3}}}{(\sqrt{3}x^{\frac{1}{2}}y\sqrt{z})^2}.$$ ***(4 marks)***

5 Factorise $3x^2 - 9x + 6$. *(2 marks)*

6 Given a function $y = f(x)$ where $f(x) = \dfrac{x-1}{x-2}$.

(i) Draw the graph of the function $y = f(x)$. *(4 marks)*

(ii) Find the domain D and the range R of the function $y = f(x)$. *(2 marks)*

(iii) Find the inverse $f^{-1}(x)$ of the function $f(x)$. *(4 marks)*

(iv) Find the domain D' and the range R' of the inverse function $y = f^{-1}(x)$. *(2 marks)*

7 Verify the following identity

$$\frac{\cos^4 x + \frac{1}{2}\sin^2(2x) + \sin^4 x}{1 + \tan^2 x} = \cos^2 x.$$ *(5 marks)*

8 Let $y = e^{x^2 + \sin x}$. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. *(4 marks)*

9 Solve the following equation for real x:

$$9^x - 3^x - 6 = 0.$$ *(5 marks)*

10 (i) Showing your working clearly, find the coefficient of x^3 in the expansion of $(1+x)^{12}$. *(2 marks)*

(ii) Use the binomial theorem to evaluate

$$\lim_{x\to\infty}\left(\sqrt{x^2 + 6x - 4} - x - 2\right).$$ *(3 marks)*

11 (i) Show that the vectors $\mathbf{u} = (3, 0, 3)$ and $\mathbf{v} = (2, 12, -2)$ are perpendicular. *(3 marks)*

(ii) A plane passes through the points $\mathbf{a} = (0, 0, 1)$, $\mathbf{b} = (1, 1, -1)$ and $\mathbf{c} = (2, -2, 1)$. Find the Cartesian equation of the plane. *(6 marks)*

12 Prove, from the definitions of $\sinh x$ and $\cosh x$, the identity

$$2\,\sinh x\,\cosh x = \sinh 2x.$$ *(3 marks)*

13 Evaluate

$$\int \coth x\,dx.$$ *(4 marks)*

14 Evaluate

$$\int \frac{2x+1}{\sqrt{x^2+4}}\,dx.$$ *(8 marks)*

15 Find the Maclaurin series for $f(x) = e^{x^2+1}$, as far as the term in x^3. *(7 marks)*

16 Complex numbers z_1 and z_2 are defined by

$$z_1 = 1 + i, \qquad z_2 = 2 - i.$$

Find, in the form $a + bi$ where a and b are real,

(i) z_1^3, *(2 marks)*

(ii) $\dfrac{z_2}{2z_1 + z_2}$. *(3 marks)*

17 Solve the system of linear equations

$$\begin{cases} x + 2y + z = 3, \\ x + 3y + 2z = 4, \\ 2x + 5y + 3z = 7. \end{cases}$$ *(5 marks)*

18 Find the volume V of the parallelepiped determined by the vectors $\mathbf{a} = (1, 1, 1)$, $\mathbf{b} = (1, 1, 0)$ and $\mathbf{c} = (2, 1, 1)$. *(4 marks)*

End of Question Paper