The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2015–2016**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5 only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below. If you attempt *more* than three only your best three will be counted.

(a) Describe the *inscription* on the Babylonian clay tablet **YBC 7289**. Where do the *cuneiform* equivalents of the *three* numbers 30, $1\,;\,24, 51, 10$ and $42\,;\,25, 35$ appear on it? Write the *second* number in *decimal* form correct to *six* decimal places. What does it represent *arithmetically*? What do the *first* and *third* numbers represent *geometrically*? Find the *third* number from the *first two* without using a calculator. ***(7 marks)***

(b) Write an entry for *Squaring the Circle* in a mathematical encyclopedia. ***(7 marks)***

(c) The years **1614, 1615, 1616, 1617, 1619, 1621, 1624** all have relevance to the early history of logarithms. For each, indicate what this relevance is, given the prompts: **1614** (... *descriptio*), **1615** (*first visit*), **1616** (Wright), **1617** (1000), **1619** (... *constructio*), **1621** (Oughtred), **1624** (base10 logarithm table). ***(7 marks)***

(d) State the *three-dimensional* version of **Cavalieri's Principle**. ***(2 marks)***

Let A be a vertical cylinder of height r with upper and lower faces circles of radius r. Let B be the cone with base A's upper face, and vertex the centre of A's lower face. Let C be the hemisphere of radius r based on A's lower face and lying in A.

For each x $(0 \leqslant x \leqslant r)$, find the areas of the horizontal cross-sections of A, B, C at height x above C's base. Deduce that the volume of the solid obtained by removing B from A is equal to that of C, and hence that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. ***(5 marks)***

[You may assume that the volume of a cone is a *third* its base *times* its height.]

2 **(a)** Outline the Egyptian (*hieroglyphic*) number system and its arithmetic. What are its main drawbacks? ***(7 marks)***

(b) **Problem 25** of the *Rhind Papyrus* reads: *A heap and its half give 16. What is the heap?*

Assume 2	$\backslash 1$	2
	$\backslash \frac{1}{2}$	1
	Total	3.

As many times as 3 must be multiplied to give 16, so many times must 2 be multiplied to give the heap.

$\backslash 1$	3
2	6
$\backslash 4$	12
$\frac{2}{3}$	2
$\backslash \frac{1}{3}$	1
Total	$5\frac{1}{3}$.

1	$5\frac{1}{3}$
$\backslash 2$	$10\frac{2}{3}$

Do it thus.

Heap is	$10\frac{2}{3}$
$\frac{1}{2}$	$5\frac{1}{3}$
Total	16.

Which script appears on the *Rhind Papyrus*? What problem is being solved here and what is this type of Egyptian problem known as? Name and explain the method that is used in its solution. Comment on aspects of Egyptian mathematics that it illustrates. ***(9 marks)***

3 The three propositions below occur in the *same* book of Euclid's *Elements*. Which book is this? How many propositions does it contain? Comment on the placement of the last two propositions within the book. ***(2 marks)***

Proposition 20 *Prime numbers are more than any assigned multitude of prime numbers.*

Proposition 35 *If numbers are in continued proportion, and there be subtracted from the second and the last of these a number equal to the first, then the excess of the second to the first equals the excess of the last to all those before it.*

Proposition 36 *If numbers, beginning with a unit, are set out in double proportion until their sum becomes prime, then this multiplied into the last is perfect.*

(a) Explain carefully the wording of **Proposition 20**. How is the result stated in modern texts? Why do you think that Euclid chose the way that he did? ***(3 marks)***

(b) Use **Proposition 35** to show that, if a and r are numbers ($a, r > 0$, $r \neq 1$), then

$$a + ar + ar^2 + \cdots + ar^{n-1} \;=\; a\,\frac{r^n - 1}{r - 1}\,.$$

Deduce that, if $2^n - 1$ is *prime*, then $2^{n-1}(2^n - 1)$ is *perfect*. Use this result, together with the fact that 8191 is prime, to find a perfect number *unknown* to the ancient Greeks. Give reasons for your answer. ***(8 marks)***

(c) How did Euler's name become linked with Euclid's formula for perfect numbers? State *two* unsolved conjectures relating to perfect numbers. ***(3 marks)***

4 Problem $\boldsymbol{\alpha}$ and **Verse $\boldsymbol{\beta}$** below are taken from the book *Quesiti et inventioni diverse*. State its author and its year of publication. Detail the personal connections that both **Problem $\boldsymbol{\alpha}$** and **Verse $\boldsymbol{\beta}$** have with the author. ***(8 marks)***

$\boldsymbol{\alpha}$ *A tree 12 units high is cut in two. The height of the tree left standing is the cube root of the length cut away. What is the height of the tree left standing?*

$\boldsymbol{\beta}$ *When the cube and the things together*
Are equal to some discrete number
Find two numbers differing in this one.
Then you will keep this as a habit
That their product should always be equal
Exactly to the cube of a third of the things.
The remainder then as general rule
Of their cube roots subtracted
Will be equal to your principal thing.

Write $\boldsymbol{\alpha}$ as a cubic equation in the height x of the tree left standing. Use $\boldsymbol{\beta}$ to show that

$$x = \sqrt[3]{6 \;+\; \sqrt{973/27}} \;-\; \sqrt[3]{-6 \;+\; \sqrt{973/27}}\,.$$

(8 marks)

5 For what purpose did Greek mathematicians employ *the method of exhaustion*? State the principle underpinning its application. Give *one* advantage and *two* disadvantages of *the method*. ***(5 marks)***

Let A and B be *consecutive* terms in a *classical exhaustion* sequence of a circle C by inscribed regular polygons. How is the vertex set of B obtained from that of A? State the inequality involving the areas of A, B, C that enables the exhaustion argument to proceed. ***(4 marks)***

In which of Archimedes' works on mensuration does he *not* use *the method of exhaustion*? What technique *did* he employ in this work? Why did he need *two* ways of establishing his results on mensuration? ***(3 marks)***

Elaborate on the following New York Times headline of July 16, 1907 :

BIG LITERARY FIND IN CONSTANTINOPLE

(4 marks)

End of Question Paper