Data provided:
Graph Paper, Laplace Transform Table

AMA312

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS Autumn Semester 2008–2009

CLASSICAL CONTROL THEORY 2 hours

Marks will be awarded for your best ***four*** *answers.*

1 (i) Using the Laplace transform method, find the output, $x(t)$, for $t \geq 0$, when the input $u(t) = t + e^{-2t}$, the initial conditions are $x(0) = 1,\ \dot{x}(0) = 0$, and the linear control system is given by the differential equation

$$\ddot{x} + 4\dot{x} + 3x = u(t)$$

(12 marks)

(ii) Find the closed-loop transfer function $H(s) = \dfrac{Y(s)}{R(s)}$ of the unity negative feedback control system with output $Y(s)$, input $R(s)$ and feedforward transfer function $C(s)G(s)$.

If $G(s) = \dfrac{n(s)}{d(s)}$ and $C(s) = \dfrac{n_c(s)}{d_c(s)}$, give the equations satisfied by the open-loop and closed-loop zeros and poles.

(5 marks)

If $C(s) = ks + \dfrac{1}{s} + 2$ in the above control system, give the differential equation satisfied by the input, $v(t)$, and output, $w(t)$, of the linear sub-system $C(s)$.

Suppose that

$$G(s) = \frac{1}{s^2 - 2s + 5}$$

Use the Routh-Hurwitz method to establish the range of gains k for which the closed-loop system with feedforward transfer function $C(s)G(s)$ is stable.

(8 marks)

2 (i) Find the impulse and step response in the time domain of the linear control system with transfer function

$$G(s) = \frac{s}{s^2 + 4s + 13}$$

Find the output, $y(t)$, for $t \geq 0$, when the input is identically zero but the initial conditions are: $y(0) = 1,\ \dot{y}(0) = -1$.

(9 marks)

(ii) Prove that

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Calculate

$$\lim_{t\to\infty} f(t)$$

for

$$F(s) = \frac{s+4}{s^2+4s+3} + \frac{2}{s(s^2+4s+3)}$$

(8 marks)

(iii) State the general form of the Routh-Hurwitz criterion.

Determine the number of stable and unstable roots of the polynomial

$$p(s) = 2s^3 + 4s^2 + 4s + 12$$

(8 marks)

3 (i) Sketch the Root Locus plot of the constant-gain feedback system with

$$G(s) = \frac{s+3}{(s^2+2s+5)(s^2+5s+4)}$$

ensuring that you compute any angles of departure and all crossover points. Give the range of gains for closed-loop stability.

(20 marks)

(ii) Using the elementary properties of the Root Locus, justify the statement: *"If the transfer function $G(s)$ has a single unstable real zero and a simple pole at 0, and all other poles are stable, then it is not stabilizable by constant gain feedback."*

(5 marks)

4 You are asked to design a constant-gain control for the unstable second-order system

$$G(s) = \frac{s+1}{(s-1)^2+1}$$

which has the following closed-loop specifications:

- The 2% settling time is no more than 10 seconds.
- The overshoot should not exceed 5%.

(i) Give the relations between settling time and the real part of the pole pair, and between overshoot and the damping ratio ζ. Sketch the region in the complex plane that must contain the closed-loop poles if the above specifications are to be met.

(13 marks)

(ii) Sketch the Root Locus and hence obtain a control gain that achieves the specifications.

(12 marks)

5 (i) Sketch the Nyquist plot (for $\omega > 0$) for

$$G(s) = \frac{20(s+1)}{(s+5)^2+1}$$

Find all crossings of the real and imaginary axes.

(12 marks)

(ii) State the general Nyquist Stability Criterion.

Sketch the Nyquist contour (for $\omega > 0$) for

$$G(s) = \frac{s+5}{s(s+1)(s+2)}$$

and find all crossings of the real and imaginary axes. Find the values of the gain k for which the closed-loop system (in the constant-gain configuration) is stable.

(13 marks)

Table of Laplace Transform Pairs

Time Function	Laplace Transform
$h(t)$	$\frac{1}{s}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$\frac{t^n e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$\cos \omega_0 t$	$\frac{s}{s^2+\omega_0^2}$
$\sin \omega_0 t$	$\frac{\omega_0}{s^2+\omega_0^2}$
$e^{-at} \cos \omega_0 t$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
$e^{-at} \sin \omega_0 t$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$

End of Question Paper