



SCHOOL OF MATHEMATICS AND STATISTICS

Spring Semester 2008-2009

FOUNDATION YEAR MATHEMATICS 1

3 hours

You should attempt all questions in Section A.

Marks will be awarded for your best three questions in Section B.

Section A carries 55 marks and each question in Section B is worth 15 marks.

Section A

- 1 (i) Make x the subject of the equation

$$\frac{3(x-1)}{2x+2} = 7y + 6. \quad (5 \text{ marks})$$

- (ii) Express

$$1 + \frac{u-v}{u+v} - \frac{v}{(u+v)^2}$$

as a single fraction, simplifying your answer where possible. (3 marks)

- 2 Simplify

$$\frac{p^3x^2 - 2xp^3y + y^2p^3}{py^2 + px^2 - 2y^2p}$$

as much as possible. (3 marks)

- 3 Simplify

$$\frac{1}{7}(24)^{\frac{2}{3}}(98)^{\frac{1}{2}}(30)^{-\frac{1}{2}}(3)^{\frac{1}{3}}$$

as much as possible. (4 marks)

4 (i) Factorize $3x^2 - 2x - 1$. (2 marks)

(ii) Solve the simultaneous equations

$$\begin{cases} 2y + x = 1 \\ 3x^2 - 2y = 3x. \end{cases} \quad (4 \text{ marks})$$

5 Simplify

$$2 \ln \left(\frac{e}{e^2 - 4} \right) + \ln(e^2 + 4e + 4) - e^{\ln 2}. \quad (5 \text{ marks})$$

6 Differentiate $f(x) = -2x^2$ from first principles. You must show your workings. (4 marks)

7 Differentiate the following functions with respect to x , simplifying your answers as much as possible.

(i) $y = x^{-\frac{1}{2}} - \frac{6}{x^2} + 2x^7$; (3 marks)

(ii) $y = e^x \cos^{-1} x$; (2 marks)

(iii) $y = \sin(\sin x)$; (2 marks)

(iv) $y = \frac{\tan x}{\ln x}$. (3 marks)

8 Find the following indefinite integrals.

(i) $\int \left(\frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} \right) dx$; (3 marks)

(ii) $\int (4x - 1)e^{2x^2 - x + 3} dx$; (3 marks)

(iii) $\int 2e^x \cos x dx$. (4 marks)

9 Evaluate the following definite integrals.

(i) $\int_0^2 \sqrt{x} dx$ (2 marks)

(ii) $\int_9^{99} \frac{1}{(x+1)^3} dx$. (3 marks)

Section B

10 Express

$$\frac{3x^4 - 3x^3 + x^2 + 3x - 2}{x^3(x^2 - 1)}$$

in partial fractions. Your answer should include a check.

(15 marks)11 (i) Find the equation of the curve $y = f(x)$ which passes through the point $(0, 2)$ and has derivative

$$\frac{dy}{dx} = 3e^{3x}. \quad (3 \text{ marks})$$

(ii) Draw the graphs of $y = x^3$ and $y = 4x$ on the same diagram. Find the area enclosed by the curves $y = x^3$ and $y = 4x$ in the quadrant with $x, y \geq 0$. **(6 marks)**

(iii) Find

$$\int \frac{1}{\sqrt{1 - (x + 2)^2}} dx. \quad (2 \text{ marks})$$

(iv) Find the volume swept out when the area under the curve $y = x^4$ enclosed by the lines $y = 0$, $x = 0$ and $x = 5$ is rotated about the x -axis through 2π radians.**(4 marks)**12 Sketch the graph of $y = \frac{e^x}{x}$.Your sketch should show clearly the coordinates and nature of all stationary points and where, if applicable, the graph crosses the x - and y -axes.(Note: you are given that $\frac{e^x}{x} \rightarrow \infty$ as $x \rightarrow \infty$.) **(15 marks)**13 (i) (a) Complete the square of $x^2 - 4x - 12$. **(2 marks)**(b) Solve $x^2 - 4x - 12 = 0$. **(2 marks)**(c) Draw the graph of $y = x^2 - 4x - 12$ and indicate clearly the minimum point of the curve and where the curve crosses the x - and y -axes. **(3 marks)**(d) Let f be the function with domain $\{x \in \mathbb{R} : 0 < x < 6\}$ and rule $f(x) = x^2 - 4x - 12$. Find the range of f . **(2 marks)**(ii) The function g has rule $g(x) = \frac{3 - \sqrt{x+2}}{x - \pi}$ and its domain is the set of all x for which $g(x)$ is defined. Find the domain of g . **(2 marks)**(iii) Let $h(x) = \frac{2x+3}{7x-4}$ for $x \neq \frac{4}{7}$. Find $h^{-1}(x)$. **(4 marks)**

14 (i) Find the quotient and the remainder when $2x^5 + x^4 - 5x^3 + 4x - 2$ is divided by $x^2 - 1$. *(10 marks)*

(ii) Using your answer to (i), or otherwise, find

$$\int \frac{2x^5 + x^4 - 5x^3 + 4x - 2}{x^2 - 1} dx$$

(5 marks)

End of Question Paper