



SCHOOL OF MATHEMATICS AND STATISTICS

**Spring Semester
2009-2010**

FOUNDATION YEAR MATHEMATICS 1

3 hours

Attempt all questions. The allocation of marks is shown in brackets.

1 Differentiate $f(x) = 10x^2$ from first principles. *(4 marks)*

2 Differentiate the following functions with respect to x :

(i) $y = 2x^5 - 3x^{\frac{1}{4}} - \frac{4}{x^7}$; *(3 marks)*

(ii) $y = (\sin x - 3x^4) \cos x$; *(2 marks)*

(iii) $y = \tan^{-1}(x^2 + 1)$; *(2 marks)*

(iv) $y = \frac{\ln x}{e^x}$. *(3 marks)*

You are expected to simplify each of your answers as much as possible.

3 Find the following indefinite integrals:

(i) $\int \left(x^3 + \frac{5}{x^3} + \frac{2}{x} \right) dx$; *(3 marks)*

(ii) $\int x \sin(2x^2 + 1) dx$; *(3 marks)*

(iii) $\int e^{2x} \sin x dx$. *(4 marks)*

4 Evaluate the following definite integrals

(i) $\int_0^{\ln 3} e^x dx;$ *(3 marks)*

(ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sin^{100}(x) dx.$ *(2 marks)*

5 Let $x = 13t^7 + 5t^3 + 1$ and $y = \frac{2}{3}t^{12} - t^3 - 1$. Find $\frac{dy}{dx}$ in terms of t . Simplify your answer as much as possible. *(3 marks)*

6 We define y implicitly as a function of x by $xy - x^2 + y^2 = 0$. Find $\frac{dy}{dx}$. *(4 marks)*

7 Let $\frac{p(x)}{q(x)} = \frac{x^7 + x^6 + x^5 + 5x^4 + x^3 - 1}{x^2(x+1)(1+x^2)}$.

(i) Express

$$\frac{x^7 + x^6 + x^5 + 5x^4 + x^3 - 1}{x^2(x+1)(1+x^2)}$$

in partial fractions. Your answer should include a check. *(18 marks)*

(ii) Using your answer to (i), or otherwise, find

$$\int \frac{x^7 + x^6 + x^5 + 5x^4 + x^3 - 1}{x^2(x+1)(1+x^2)} dx.$$

(8 marks)

8 Sketch the graph of $y = f(x) = \frac{1}{x^4} + x^4$.

Your sketch should show clearly the coordinates and nature of all stationary points and where, if applicable, the graph crosses the x - and y -axes. *(15 marks)*

9 Find the equations of the tangent and the normal to the curve $y = \frac{1}{x} - x$ at the point $\left(2, -\frac{3}{2}\right)$. *(11 marks)*

- 10** (i) Sketch the graph of $y = x^2 + x - 6$. *(4 marks)*
- (ii) Find the area enclosed by the curve $y = x^2 + x - 6$, the x -axis and the lines $x = 1$ and $x = 3$. *(3 marks)*
- 11** Find the volume swept out when the area under $y = x^2 - 4$ between $x = 2$ and $x = 4$ is rotated about the x -axis through 2π radians. *(5 marks)*

End of Question Paper