Data provided: Formula sheet

MAS157

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2009–2010**

Mathematics for Chemists **2 hours**

There are 50 marks in Section A and 50 marks in Section B, so you are advised to spend about 1 hour on Section A and about 1 hour on Section B. The marks awarded to each question or section of question are shown in italics.

Section A

A1 Showing your working clearly, find

(a) The coefficient of x^4 in the expansion of $(1+x)^8$. ***(3 marks)***

(b) The coefficient of x^2 in the expansion of $(1+x)^{37}$. ***(3 marks)***

A2 Use the binomial theorem to evaluate

$$\lim_{x\to\infty}\left\{(x^3+x^2-1)^{1/3}-x\right\}.$$ ***(8 marks)***

A3 Vectors **a**, **b** and **c** are given by

$$\mathbf{a}=(1,1,2),\qquad \mathbf{b}=(3,-1,0),\qquad \mathbf{c}=(-1,2,-1).$$

(a) Find the angle, in radians correct to two decimal places, between **a** and **b**. ***(5 marks)***

(b) Verify that $\mathbf{b}\times\mathbf{c}$ is perpendicular to both **b** and **c**. ***(6 marks)***

A4 From the definitions

$$\sinh x=\frac{1}{2}\left(e^x-e^{-x}\right)\qquad\text{and}\qquad \cosh x=\frac{1}{2}\left(e^x+e^{-x}\right)$$

show that $\cosh^2 x - \sinh^2 x = 1$. ***(5 marks)***

A5 (a) By using $\tanh x = \dfrac{\sinh x}{\cosh x}$, show that

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x.$$ ***(4 marks)***

(b) If $y = \ln(\cosh x)$, show that

$$\frac{dy}{dx} = \tanh x.$$ ***(2 marks)***

(c) If $y = \dfrac{\sinh 2x}{\sinh x}$, show that

$$\frac{dy}{dx} = 2\sinh x.$$ ***(6 marks)***

A6 Complex numbers z_1 and z_2 are defined by

$$z_1 = 2 + 3i \qquad \text{and} \qquad z_2 = 1 - i.$$

Find, in the form $a + bi$ where a and b are real,

(a) z_2^4 ***(3 marks)***

(b) $\dfrac{z_1}{z_1 + z_2}$ ***(5 marks)***

Section B

B1 (a) By making the substitution $x = a\cosh\theta$, show that

$$\int_a^{2a} \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} 2,$$

where a is a positive constant. ***(8 marks)***

(b) Show that the Maclaurin series for $\sinh x$ is

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$ ***(6 marks)***

(c) A plane passes through the points $(-1, 3, 0)$, $(1, 0, 1)$ and $(2, 1, 0)$.

Find two vectors which lie in the plane, and hence find a vector normal to the plane. ***(7 marks)***

Show that the Cartesian form of the equation of the plane is

$$2x + 3y + 5z = 7.$$ ***(4 marks)***

B2 (a) Matrices A and B are defined by

$$A = \begin{pmatrix} -1 & 3 \\ 2 & -4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}.$$

(i) Find the inverses, A^{-1} and B^{-1}, of A and B, respectively. ***(6 marks)***

(ii) Verify that $(AB)^{-1} = B^{-1}A^{-1}$. ***(7 marks)***

(b) A set of linear equations can be written as

$$D \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix},$$

where

$$D = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -1 & 1 \end{pmatrix}.$$

Find the inverse, D^{-1}, of D, and use it to find the values of x, y and z which satisfy the equations. ***(12 marks)***

End of Question Paper