



SCHOOL OF MATHEMATICS AND STATISTICS

Autumn Semester
2010–2011

Foundation Year Mathematics I

1 hour 30 minutes

Attempt *all* questions. The allocation of marks is shown in brackets.

- 1 Express

$$1 + \frac{a}{a+b} - \frac{a}{a-b}$$

as a single fraction, simplifying your answer where possible.

(3 marks)

- 2 Simplify

$$\frac{3s^6z^3 - 3t^8z^3}{6s^6z^4 - 12s^3t^4z^4 + 6t^8z^4}$$

as much as possible.

(3 marks)

- 3 Simplify

$$(50)^{\frac{1}{3}}(27)^{\frac{1}{2}}(20)^{-\frac{1}{3}}(135)^{-\frac{1}{3}}$$

as much as possible. Show all your working, which should include using laws of indices.

(4 marks)

- 4 Rationalize the denominator of $\frac{1 - \sqrt{5}}{2 + 3\sqrt{5}}$.

(3 marks)

5 (i) Factorize $6x^2 - x - 2$. *(2 marks)*

(ii) Solve the simultaneous equations

$$\begin{cases} y - xy - 1 = 0 & (1) \\ 6x^2 + \frac{1}{y} - 3 = 0 & (2). \end{cases}$$

Your solution should include a check. *(7 marks)*

6 (i) Complete the square for $x^2 - x - 4$. *(2 marks)*

(ii) Solve $x^2 - x - 4 = 0$. *(3 marks)*

(iii) Draw the graph of $y = x^2 - x - 4$ and indicate clearly the minimum point of the curve and where the curve crosses the x - and y -axes. *(4 marks)*

(iv) Let f be the function with domain $\{x \in \mathbb{R} : -3 < x \leq 3\}$ and rule $f(x) = x^2 - x - 4$. Find the range of f . *(2 marks)*

7 Let $p(x) = 3x^3 - 2x^2 - 2x + 1$, $q(x) = 2x^2 - 4x - 7$ and $r(x) = -x^2 - 4$.

(i) Expand $p(x) - q(x)r(x)$, collecting like terms together. What is the coefficient of x^3 in $p(x) - q(x)r(x)$? *(6 marks)*

(ii) Find $q(r(x))$. *(3 marks)*

8 Express the following fractions as partial fractions. Your answer should include a check.

(i) $\frac{7}{(2x+3)(x-2)}$. *(9 marks)*

(ii) $\frac{x^6 + x^4 - x^3 + 3x^2 - 2x + 1}{x^2(x^2 + 1)}$. *(16 marks)*

9 Let f be the function with rule $f(x) = \frac{x-3}{6-x} - 2$ and domain $\{x \in \mathbb{R} : x \neq 6\}$.

(i) Find the inverse function of $f(x) = \frac{x-3}{6-x} - 2$. *(8 marks)*

(ii) What is the domain of f^{-1} ? *(1 mark)*

(iii) State the range of f . *(1 mark)*

10 Sketch the graphs of the following functions between 0 and 2π

(i) $y = \cos(x)$. *(3 marks)*

(ii) $y = \cos(2x)$. *(4 marks)*

(iii) $y = -3\cos(x)$. *(4 marks)*

11 Simplify

$$\frac{1}{2} \ln(e^4 - 2e^3 + e^2) - \ln(e^6 - e^4). \quad (4 \text{ marks})$$

You should show all your working.

12 Solve the following equation for x , giving your answer in terms of $\ln 2$ and $\ln 3$. Your answer must be **exact**, so do not use a calculator.

$$\frac{2^{x+1}}{9^{x-1}} = \frac{e^2}{3^{3x-2}}.$$

(8 marks)

End of Question Paper



SCHOOL OF MATHEMATICS AND STATISTICS

Spring Semester
2010-2011

FOUNDATION YEAR MATHEMATICS 1

3 hours

Attempt *all* questions. The allocation of marks is shown in brackets.

1 Differentiate $f(x) = 2x^2 - 1$ from first principles. (4 marks)

2 Differentiate the following with respect to x .

(i) $y = x^7 + 5\sqrt{x} + \frac{9}{x^{\frac{5}{3}}} - 5;$

(ii) $y = \left(\frac{2}{5}x^5 + x^3\right) e^x;$

(iii) $y = \sin(2x^4 + 3x^5);$

(iv) $y = \frac{\ln x}{\tan^{-1} x}.$ (11 marks)

You are expected to simplify each of your answers as much as possible.

3 Find the following indefinite integrals.

(i) $\int \left(6x^7 + \frac{9}{x^8} - 4\sqrt[3]{x}\right) dx;$

(ii) $\int (x - 2) \cos(x^2 - 4x) dx;$

(iii) $\int (3x - 1)e^x dx .$ (8 marks)

4 Evaluate the following definite integrals.

(i) $\int_1^8 x^{\frac{1}{3}} dx;$

(ii) $\int_0^\pi \sin(2x - \pi) dx.$ *(5 marks)*

5 Using parametric differentiation, find $\frac{dy}{dx}$ in terms of t when $y = \sin^{-1} t$ and $x = \sqrt{1 - t^2} + 2.$ *(3 marks)*

6 A curve has equation $5y^3 - 3xy^2 + x = 4.$ Find $\frac{dy}{dx}.$ *(4 marks)*

7 (i) Express

$$\frac{1}{1 - x^3}$$

as partial fractions. Your answer should include a check. *(13 marks)*

(ii) Given that $\int \frac{1}{x^2 + x + 1} dx = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) + c,$ find

$$\int \frac{1}{1 - x^3} dx.$$
 (5 marks)

8 (i) Find the stationary points of the curve

$$y = f(x) = 7x^3 - 3x^7$$

and determine their nature. *(9 marks)*

(ii) Sketch the graph of $y = f(x) = 7x^3 - 3x^7.$ *(8 marks)*

9 Find the volume swept out when the area under $y = 2x + 1$ enclosed by $y = 0,$ $x = 0$ and $x = \frac{1}{2}$ is rotated about the x -axis through 2π radians. *(5 marks)*

10 Find the area enclosed by the curves $y = 11x^2 + 13x - 12$ and $y = -7x^2 - 5x + 24.$ *(7 marks)*

- 11** At time t , the length of a side of a cube is l mm. The length is increasing at 2 mm/sec. Find the rate at which the volume of the cube is increasing when $l = 10$ mm. *(6 marks)*

- 12** Find the equation of the curve $y = f(x)$ which passes through the point $(0, 2)$ and has derivative

$$f'(x) = 2xe^{x^2}. \quad (3 \text{ marks})$$

- 13** Find the derivative of

$$y = \ln \left(\frac{(x^4 \sin x^2)^3 ((x+1)^7 \tan^{-1} x)}{e^x \ln x} \right). \quad (9 \text{ marks})$$

End of Question Paper