Data provided: Formula sheet

MAS157

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2010–2011**

Mathematics for Chemists **2 hours**

There are 50 marks in Section A and 50 marks in Section B, so you are advised to spend about 1 hour on Section A and about 1 hour on Section B. The marks awarded to each question or section of question are shown in italics.

Section A

A1 Showing your working clearly, find

(a) the coefficient of x^3 in the expansion of $(1+x)^9$; *(3 marks)*

(b) the coefficient of x^2 in the expansion of $(1+x)^{43}$. *(3 marks)*

A2 Use the binomial theorem to evaluate

$$\lim_{x\to 0} \frac{(8+x)^{1/3} + (8-x)^{1/3} - 4}{x^2}. \qquad \textit{(11 marks)}$$

A3 Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by

$$\mathbf{a} = (3,1,1), \quad \mathbf{b} = (1,2,-1), \quad \mathbf{c} = (1,0,4).$$

Find

(a) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$ *(4 marks)*

(b) $\mathbf{b}\times(\mathbf{a}\times\mathbf{c})$ *(4 marks)*

A4 Let $y = \sinh^{-1} x$ and $z = e^y$.

Show that

$$z^2 - 2xz - 1 = 0,$$

and hence show that

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right). \qquad \textit{(8 marks)}$$

A5 If $x = 1 - y^2$ for $y \geqslant 0$, find dy/dx as a function of x. *(4 marks)*

A6 By making the substitution $u = \sinh x$, evaluate

$$\int \coth x \, dx. \qquad \textit{(4 marks)}$$

A7 A damped pendulum makes an angle θ with the vertical, where θ satisfies the differential equation

$$\frac{d^2\theta}{dt^2} + 2\frac{d\theta}{dt} + 5\theta = 0,$$

with $\theta = 2$ and $d\theta/dt = 0$ when $t = 0$.

Verify that

$$\theta = e^{-t}(2\cos 2t + \sin 2t)$$

is a solution to this problem. *(9 marks)*

Section B

B1 (a) Evaluate

$$\int \frac{x^2+3}{(x-1)(x^2+5x+6)}\,dx. \qquad \textit{(13 marks)}$$

(b) Show that the Maclaurin series for $\ln(1-x)$ is

$$\ln(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\cdots \qquad \textit{(8 marks)}$$

Use the ratio test to show that this series converges for $|x| < 1$. *(4 marks)*

B2 (a) Find the two values of α for which the set of equations

$$\begin{array}{rcrcrcl} x & + & y & + & \alpha z & = & 0 \\ 2x & & & + & (1-\alpha)z & = & 0 \\ \alpha x & - & y & - & z & = & 0 \end{array}$$

has a non-trivial solution. *(5 marks)*

Find the solutions corresponding to each of these two values of α. *(6 marks)*

(b) Use de Moivre's theorem to show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta. \qquad \textit{(7 marks)}$$

(c) On an Argand diagram, sketch the locus given by

$$|z-1|^2+|z+1|^2 = 4. \qquad \textit{(7 marks)}$$

End of Question Paper