Data provided: Formula sheet

**MAS165**

The University Of Sheffield.

**SCHOOL OF MATHEMATICS AND STATISTICS** **Spring Semester 2010-2011**

**Mathematics for Physicists** **2 hours**

*This paper comprises TWO sections. You should attempt ALL questions on this paper.*

## Section A

**A1** You are given the points A = (1, 2, 3), B = (1, 3, 2) and C = (4, 3, 2).

(i) Find the vector equation of the plane passing through A, B and C. ***(5 marks)***

(ii) Using (i), or otherwise, find the perpendicular distance of the plane from the origin O = (0, 0, 0). ***(4 marks)***

(iii) A line is given by $\mathbf{r} = \mathbf{p} + t\mathbf{q}$, where the vectors $\mathbf{p}$ and $\mathbf{q}$ are given as $\mathbf{p} = (0, 1, 0)$ and $\mathbf{q} = (3, 4, 0)$. Determine the angle between this line and the plane going through the points $A$, $B$ and $C$. ***(4 marks)***

**A2** (i) Show that $(\mathbf{c} \times (\mathbf{b} \times \mathbf{c})) \times \mathbf{c} = c^2\mathbf{b} \times \mathbf{c}$, where $c$ is the magnitude of the vector $\mathbf{c}$. ***(3 marks)***

(ii) If $\mathbf{r} = (1, 1, 1)$, $\mathbf{s} = (2, 0, 3)$ and $\mathbf{t} = (0, 1, 3)$, find $\mathbf{r} \times \mathbf{s}$ and $\mathbf{s} \times \mathbf{t}$ and hence show that

$$\mathbf{r} \cdot (\mathbf{s} \times \mathbf{t}) = (\mathbf{r} \times \mathbf{s}) \cdot \mathbf{t}$$

***(5 marks)***

**A3** Stokes' theorem may be written:

$$\oint_C \mathbf{G}.\mathbf{dr} = \int_S (\nabla \times \mathbf{G}).\hat{\mathbf{n}}\, dS$$

Indicate whether the following statements about Stokes' theorem, as expressed here, are true or false

(i) The term $(\nabla \times \mathbf{G})$ is the curl of the vector field $\mathbf{G}$.

(ii) The surface $S$ is surrounded by a closed line $C$.

(iii) $\hat{\mathbf{n}}$ is a unit vector parallel with the boundary $C$.

(iv) $\int_S dS$ is a surface integral, over the surface $S$.

***(4 marks)***

## Section B

**B1** (i) Let $f(t) = t^2 - 3$ be a scalar function and $\mathbf{A}(t) = (t, e^t, -t^2)$, $\mathbf{B}(t) = (\cos 2t, \sin 2t, 0)$ and $\mathbf{C}(t) = (t, t^2, -3t)$ are vectors. Calculate $d(f\mathbf{A})/dt$, $d(\mathbf{B} \cdot \mathbf{C})/dt$ and $d(\mathbf{B} \times \mathbf{C})/dt$. ***(9 marks)***

(ii) A scalar field is given by

$$\phi = x^3 + xz + yz + 3$$

Find the gradient of $\phi$, and hence find the directional derivative of $\phi$ at the point $(1, 2, 3)$ in the direction $(1, 2, 0)$. ***(4 marks)***

(iii) Sketch the region of integration represented by the repeated integral

$$\int_0^1 \int_{2y}^2 (2x + y)dxdy$$

Evaluate the integral with the order of the integration reversed.

***(12 marks)***

**B2** A force is given by $\mathbf{F} = xy^2\mathbf{i} + 2\mathbf{j} + x\mathbf{k}$ and $L$ is the path of a particle, specified by the vector $\mathbf{r} = (x, y, z)$ with $x = ct$, $y = c/t$ and $z = d$, where $c$ and $d$ are positive constants and $1 \leq t \leq 2$. Consider the following integral

$$C = \int_L \mathbf{F} \cdot d\mathbf{r}$$

(i) Does this integral result in a vector or in a scalar? ***(2 marks)***

(ii) Evalulate the integral $C$. ***(15 marks)***

(iii) Calculate the divergence and the curl of $\mathbf{F}$ and show that the curl is constant on the path $L$. ***(8 marks)***

**B3** (i) The surface density of a circular disc of radius $a$ is $\sigma_0 r^2/a^2$, where $\sigma_0$ is a positive constant. Find the mass of the disc. ***(8 marks)***

(ii) Find the tangent plane and normal at the point $(1, 1, 1)$ for the surface specified by

$$x^3 + y^3 + z^3 = 3.$$

***(5 marks)***

(iii) Sketch the region of integration represented by the integral

$$\int\int_{\mathcal{R}} (x^2 + y^2)^{1/2} dx dy,$$

where $\mathcal{R}$ is the circular sector such that $x^2 + y^2 \leq a^2$, $x \geq 0$ and $y \geq 0$. Evaluate the integral by transforming to plane polar coordinates.

***(12 marks)***

## End of Question Paper