Data provided: formula sheet

The University Of Sheffield.

MAS252

SCHOOL OF MATHEMATICS AND STATISTICS **Autumn Semester 2010–11**

Further Civil Engineering Mathematics and Computing **2 hours**

Answer ***four*** *questions. You are advised* ***not*** *to answer more than four questions: if you do, only your best four will be counted.*

1 (i) Write down the first 4 terms in the binomial series of $\sqrt{9-x}$. Use this result to evaluate $\sqrt{7.5}$ with a precision of 2 decimal places. ***(9 marks)***

(ii) Use the L'Hopital's rule to find

$$\lim_{x\to 0}\left(\frac{1}{x^2+x}-\frac{1}{\sin x}\right).$$

(9 marks)

(iii) Use the Newton-Raphson scheme defined as

$$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)},\quad k=0,1,2\ldots,$$

to find the root of the function

$$f(x)=\ln\left(\sqrt{x}+1\right)-x^2,\qquad x\geq 0$$

using $x_0=2$ as an initial guess. Work correct to 4 decimal places. ***(7 marks)***

2 (i) The function $y(x)$ satisfies the ordinary differential equation

$$\frac{dy}{dx}=x^2+\sin y+1$$

and the initial condition $y(0)=0$. Determine the Maclaurin series solution of this equation as far as the term in x^3. Use this series to evaluate $y(0.2)$, giving your answer correct to 3 decimal places. ***(10 marks)***

 Question 2 continued on next page

2 (continued)

(ii) The function $f(x, y)$ is defined as

$$f(x, y) = \ln(\sqrt{x^2 + y^2}) + \sin\left(\frac{x^2}{y^2}\right).$$

Calculate the derivates $\partial f/\partial x$, $\partial f/\partial y$, $\partial^2 f/\partial x \partial y$ and $\partial^2 f/\partial x^2$.

(15 marks)

3 (i) The temperature response function of an engine is given by the function

$$f(\alpha, T) = \frac{\ln(1 + \sqrt{\alpha^2 + T^2})}{\alpha}.$$

According to the designer specification, the engine works under ideal conditions for $T = 110$ and $\alpha = 45$, measured in some appropriate units. During its use it is observed that the quantity T has a change of ± 45 while the quantity α has a change of ± 7. Given these possible changes, use the small increment law to find the change in the temperature response function, f. Work correct to five decimal places. ***(10 marks)***

(ii) Use the chain rule to evaluate dz/dt at $t = \pi/2$ given that $z(x, y) = \sin(x/y)$ where $x(t) = t - \cos t$ and $y = \sin t/t$ giving your answer correct to *three* decimal places. ***(15 marks)***

4 (i) A certain differential equation is solved up to $x = 1$ using the fourth-order Runge-Kutta method firstly with $h = 0.1$ and secondly with $h = 0.2$ and we find the values given in the attached table, where $Y(x)$ are the computed values of $y(x)$. Use this data to estimate the value of h which will ensure an accuracy of the result (in absolute value) of *four* decimal places when calculating $y(1)$ using the same method.

h	Y(1)
0.1	15.22196
0.2	15.21117

Hint: the global error in the case of the fourth-order Runge-Kutta method is calculated using

$$y(x) - Y(x) \approx Ch^4$$

where C is a constant ***(10 marks)***

 Question 4 continued on next page

4 (continued)

(ii) Show that the trigonometric Fourier series of $f(x) = 3x+1$ for $-\pi \leq x \leq \pi$ is given by

$$f(x) = 1 - 6\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx.$$

(15 marks)

5 (i) Find the solution of the heat conduction equation

$$4\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u = u(x,t), \quad 0 \leq x \leq 2, \quad t \geq 0$$

in the separable form $u(x,t) = X(x)T(t)$, subject to the boundary and initial conditions

$$u(0,t) = 0, \quad t \geq 0,$$

$$u(2,t) = 0 \quad t \geq 0,$$

$$u(x,0) = 2\sin\left(\frac{\pi x}{2}\right) - \sin(\pi x) + 4\sin(2\pi x) = f(x).$$

Hint: the orthogonality of the sine function can be defined as

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right) dx = \begin{cases} 0 & \text{if } m \neq n \\ L/2 & \text{if } m = n \end{cases}$$

(25 marks)

End of Question Paper