The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2010–2011**

Mathematics (Computational and Numerical Methods) **2 hours**

Answer ***four*** *questions. You are advised* ***not*** *to answer more than four questions: if you do, only your best four will be counted.*

1 (i) Let $f(x) = x^2 - a$. Show that the Newton-Raphson method for finding the root of $f(x)$ leads to the recurrence equation

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

(4 marks)

(ii) Using the Newton-Raphson formula, find the root of the function $f(x) = 0.5 + \sin(x) - \sin(2x)$, starting from $x_0 = -\pi/2$, with a precision of 3 decimal places.

Hint: $\sin(-x) = -\sin(x)$, $\cos(-x) = \cos(x)$. ***(7 marks)***

(iii) Use the Jacobi and Gauss-Seidel iterative methods to compute two successive approximations to the solution of the system of equations

$$\begin{aligned} 9x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 10x_2 + 3x_3 &= 3 \\ 3x_1 + 4x_2 + 11x_3 &= 4 \end{aligned}$$

starting with the initial guess $x_0 = [0, 0, 0]^T$. Give your answer correct to four decimal places. ***(12 marks)***

Do you expect convergence when the Jacobi and Gauss-Seidel iterative methods are applied to the above system? Explain! ***(2 marks)***

2 (i) Use Gaussian elimination, with partial pivoting, to solve the system of equations

$$\begin{aligned} 8x_2 + 2x_3 &= -7 \\ 3x_1 + 5x_2 + 2x_3 &= 8 \\ 6x_1 + 2x_2 + 8x_3 &= 26 \end{aligned}$$

(10 marks)

(ii) Factorise the matrix

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -4 & 6 \\ 1 & 3 & 4 \end{pmatrix}$$

into the product $A = LU$ where L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. Hence using this factorisation, solve for $\mathbf{x}$

$$A\mathbf{x} = \mathbf{b}$$

where $\mathbf{x} = [x_1, x_2, x_3]^T$ and $\mathbf{b} = [14, 34, 6]^T$. ***(15 marks)***

3 (i) It is suspected that the data

x	-1	0	1	2	3
y	0.52	1.03	2.03	4.01	7.92

can be represented by $y = ae^{bx}$ where a and b are constants. By employing a suitable transformation and then using the least square linear fit, find the least square values of a and b correct to two decimal places. ***(15 marks)***

(ii) The value of π can be found using the integral

$$\pi = 4\int_0^1 \frac{1}{1+x^2}dx$$

Obtain the value of π up to four decimal places using Simpson's rule with a step size of 0.1. ***(10 marks)***

4 (i) Find all the eigenvalues and associated eigenvectors for the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

(12 marks)

(ii) Use the Lagrange's interpolating formula to find the interpolating polynomial P_3 through the points (0,3), (1,2), (2,7), (4,59), and then find the approximate value of $P_3(3.1)$ giving your answer correct to three decimal places. ***(13 marks)***

5 (i) Using a graphical approach, maximise the function

$$f(x_1, x_2) = 5x_1 + 6x_2$$

subject to the conditions

$$x_1 + x_2 \leq 10$$

$$x_1 - x_2 \geq 3$$

$$5x_1 + 4x_2 \leq 35$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Calculate the equation of the iso-profit line and draw it on the graph.

(13 marks)

(ii) A company manufactures two products (A and B) and the profit per unit sold is £3 and £5 respectively. Each product has to be assembled on a particular machine, each unit of product A taking 12 minutes of assembly time and each unit of product B requiring 25 minutes of assembly time. The company estimates that the machine used for assembly has an effective working week of only 30 hours (due to maintenance/breakdown). Technological constraints mean that for every five units of product A produced at least two units of product B must be produced. The aim of the company is to have maximum profit. Formulate the problem as a linear programming programme and solve the problem graphically. Calculate the equation of the iso-profit line and draw it on the graph. ***(12 marks)***

End of Question Paper