



SCHOOL OF MATHEMATICS AND STATISTICS

Spring Semester 2010–2011

History of Mathematics

2 hours 30 minutes

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5, only your best three will be counted.

1 Attempt *three* of questions (a), (b), (c), (d) below.

(a) List the *three* ancient Egyptian scripts in chronological order. For each, name a primary source, *not* a papyrus, on which it occurs, giving *three* different answers. Why do Egyptologists habitually transcribe the original script on a papyrus to another script? (6 marks)

(b) Show that the *third* hexagonal number is 15, and determine the *fourth*. Given $2^2 \times 17 \times 43$ is *one* number of an amicable pair, show the *other* is $2^2 \times 5 \times 131$. (6 marks)

(c) The concluding words of a mathematical classic are (in translation):

WRITTEN IN FIVE YEARS, MAY IT LAST AS MANY THOUSANDS

Identify the work, its author, year of publication and the language in which it was written. Mention *four* of its mathematical highlights. (6 marks)

(d) State the principle underpinning the Greek *method of exhaustion*. In which book does Archimedes use *this* method to prove a *formula* for the area of a parabolic segment? State the *formula* and use it to show $\int_{-a}^a x^2 dx = \frac{2}{3}a^3$ ($a > 0$). (6 marks)

2 Describe the *Old Babylonian* number system. What do you see as its strengths and its weaknesses? Comment on a development of the number system that occurred in the early years of the Seleucid period. (9 marks)

What inscription is on the Babylonian clay tablet **YBC 7289**? Where do the *cuneiform* equivalents of the *three* numbers 30, 1; 24, 51, 10, 42; 25, 35 appear on it? What does the *second* number represent *arithmetically*? Write it in *decimal* form correct to *six* decimal places. What do the *first* and *third* numbers represent *geometrically*? State how the *three* numbers are related *algebraically*, and obtain the *second* from the *first* and *third*. What can be learned of Babylonian, arithmetic, geometry and algebra from **YBC 7289**? (9 marks)

3 Outline the structure and contents of Euclid's *Elements* Book I. Illustrate your answers with *specific* examples. **(11 marks)**

What do you see as its shortcomings as: (a) a student textbook; (b) a rigorous mathematical treatise? **(7 marks)**

4 Below are titles of books by *three* British mathematicians. For each book, name its author and indicate its contents.

A Briefe and True Report of the New Found Land of Virginia

A Plaine Discovery of the Whole Revelation of Saint John

The Urinal of Physick

(4 marks)

Which *two* of the authors you named above, published *mathematics* books during their lifetimes? For each, choose the title of *one* such book, say what motivated its publication, suggest reasons for the choice of language in which it was written, and comment on its success. **(8 marks)**

Which author published *no* mathematics book during his lifetime? Suggest reasons for this. What has happened to his mathematical papers since his death? **(6 marks)**

5 (a) Define a *cycloid*. Sketch (as in the course notes) Roberval's proof that the area under a cycloidal arch is *three* times that of its generating circle. Which celebrated curve makes its first known appearance in this proof? **(9 marks)**

(b) Summarize Fermat's contributions to the early calculus. **(3 marks)**

Show how he would have found the area under the curve $y = \sqrt{x}$ and above the x -axis from 0 to a , where $a > 0$. You may assume that, for $0 < E < 1$,

$$\frac{1 - E}{1 - E^{3/2}} = \frac{1 + \sqrt{E}}{1 + \sqrt{E} + E}. \quad \text{(6 marks)}$$

End of Question Paper