



SCHOOL OF MATHEMATICS AND STATISTICS

May 2011

MAS5050: Mathematical Methods for Statistics

2 hours

RESTRICTED OPEN BOOK EXAMINATION

Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.

Candidates should attempt ALL questions.

The maximum marks for the various parts of the questions are indicated.

The paper will be marked out of 80.

1 When a coin is tossed, it produces either a result of heads (H) or tails (T). Let Ω be the set of possible outcomes from tossing a coin four times, that is

$$\Omega = \{(a, b, c, d) \mid a, b, c, d \in \{H, T\}\}.$$

Let A be the subset of Ω representing those outcomes where precisely one coin toss is a head, and let B be the subset representing those outcomes where at least two coin tosses are tails.

- (i) Write A and B in list form (ie: as a list of elements enclosed in $\{\dots\}$).
(3 marks)
- (ii) Write $A \cup B$, $A \cap B$, and $B \setminus A$ in list form.
(3 marks)
- (iii) Describe the set $\Omega \setminus B$ in words.
(2 marks)

- 2 (i) Find constants A and B such that

$$\frac{2}{(n+1)(n+3)} = \frac{A}{n+1} + \frac{B}{n+3}.$$

(3 marks)

- (ii) Let $N \geq 4$. Find the sum

$$\sum_{n=1}^N \frac{2}{(n+1)(n+3)}.$$

(7 marks)

- (iii) Find the sum

$$\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)}.$$

(2 marks)

- 3 (i) Differentiate the following functions with respect to x :

(a) $f(x) = \sin(x^3).$

(2 marks)

(b)

$$g(x) = \frac{e^x \cos x}{1+x^2}.$$

(4 marks)

- (ii) Let $h(x) = 3x - 4x^3$. Find the maximum and minimum value of $f(x)$ when $0 \leq x \leq 1$. (6 marks)

- 4 Evaluate the following integrals.

(i)

$$\int_e^{e^2} \frac{1}{x \log_e x} dx.$$

(6 marks)

(ii)

$$\int_e^{e^2} x \log_e x dx.$$

(6 marks)

- 5 (i) Calculate the Jacobian of the transformation

$$x = 2r \cos \theta \quad y = r \sin \theta.$$

(5 marks)

- (ii) Let A be the region

$$\{(x, y) \mid 1 \leq x^2 + 4y^2 \leq 4\}.$$

Evaluate the double integral

$$\int \int_A \frac{1}{x^2 + 4y^2} dx dy.$$

(8 marks)

- 6 Use Gaussian elimination to solve the set of linear equations

$$\begin{aligned} x + 2y + 5z &= 9 \\ 2x + y + 5z &= 7 \\ x + 2y + 3z &= 5 \end{aligned}$$

(8 marks)

- 7 Let

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}.$$

- (i) Find the matrix product AB . (3 marks)
- (ii) Find the matrix inverse of B . (3 marks)
- (iii) Find a matrix P and a diagonal matrix D such that $P^{-1}AP = D$. (9 marks)

End of Question Paper