



The  
University  
Of  
Sheffield.

**SCHOOL OF MATHEMATICS AND STATISTICS**

**Spring Semester  
2010–2011**

**Probability and Probability Distributions**

**2 hours**

*RESTRICTED OPEN BOOK EXAMINATION.*

*Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.*

*Candidates should attempt **All** questions.*

*The maximum marks for the various parts of the questions are indicated.*

*The paper will be marked out of 80.*

*PDF stands for Probability Density function and CDF for Cumulative Distribution Function.*

**Please leave this exam paper on your desk  
Do not remove it from the hall**

Registration number from U-Card (9 digits)  
to be completed by student

--	--	--	--	--	--	--	--	--

**Blank**

**1** (i) Let  $T$  be a random quantity distributed according to a  $t$  distribution with 4 degrees of freedom.

(a) Find the value of  $y$ , such that  $P[T < y] = 0.01$  *(2 marks)*

(b) Use tables to find bounds for  $P[T > 1.9]$  *(2 marks)*

(ii) Let  $F$  be distributed according to an F distribution with 9 and 5 degrees of freedom.

(a) Find the value of  $y$  such that  $P[F < y] = 0.9$  *(2 marks)*

(b) Use tables to find bounds for  $P[F > 0.1]$  *(2 marks)*

**2** Let  $X \in (-1, 1)$  be a random quantity with PDF given by

$$f(x) = \begin{cases} x + 1 & -1 \leq x < 0 \\ 1 - x & 0 \leq x \leq 1 \end{cases}$$

(i) Sketch  $f(x)$  *(1 mark)*

(ii) Prove that  $f(x)$  is a pdf *(3 marks)*

(iii) Calculate  $E[X]$  and  $\text{Var}[X]$  *(4 marks)*

(iv) Calculate the CDF of  $X$  *(5 marks)*

(v) Calculate  $P[X > 1/2 \mid X > 0]$  *(3 marks)*

**3** Assume that  $U$  is a continuous quantity which follows a uniform distribution on  $(0, 1)$ .

(i) Find the distribution of  $X = \log \frac{1}{U}$  *(5 marks)*

(ii) For  $t > 0$  and  $a > 0$ , calculate  $P[X > t + a \mid X > a]$  *(3 marks)*

**4** Suppose a typist commits on average one error every two pages. Assuming that the number of errors per page follows a Poisson distribution:

(i) What is the probability of him typing a flawless booklet of 7 pages? *(2 marks)*

(ii) The booklet will go to print if no more than four errors are found. What is the probability of sending the booklet back for revisions? *(3 marks)*

(iii) After obtaining new software, the typist's average accuracy increases to one error in 20 pages. If he is commissioned to type a book containing 1000 pages, find an approximate probability of the book containing more than 50 errors. *(3 marks)*

**5** Consider the bivariate function  $f_{XY}(x, y) = K(x + y)$ , defined on the unit square  $S = \{(x, y) : 0 < x < 1, 0 < y < 1\}$

- (i) Find the value of  $K$  such that  $f_{XY}(x, y)$  will be a joint PDF? *(3 marks)*
- (ii) If  $X$  and  $Y$  are random variables with joint PDF as in (i), find  $E[XY]$  *(4 marks)*
- (iii) Find the marginal distributions of  $X$  and  $Y$  and calculate  $E[X]$  and  $E[Y]$  *(6 marks)*
- (iv) Calculate  $\text{Cov}[X, Y]$ . Are  $X$  and  $Y$  independent? *(3 marks)*

**6** Use Neave's table to calculate the following. For (i) also provide the  $R$  command you would use to get an exact answer.

- (i)  $P[X > 23]$ 
  - (a) if  $X \sim N(16, 16)$  *(1 mark)*
  - (b) if  $X \sim N(25, 1/4)$  *(1 mark)*
- (ii) If  $X \sim N(-2, \sigma^2)$ , find  $\sigma^2$  such that  $P[|X + 2| < 4] = 0.95$  *(4 marks)*
- (iii) Approximate  $P[0 < X \leq 9]$  if  $X \sim \text{Bi}(14, 0.33)$ . *(2 marks)*

**7** Consider the experiment of tossing two tetrahedra (regular four-sided "dice"), with their sides marked 1,2,3,4. Let  $X$  denote the number showed by the first tetrahedron and  $Y$  the larger of both numbers shown.

- (i) Find the support (sample space) and the joint distribution of  $X$  and  $Y$ . *(3 marks)*
- (ii) Find the joint CDF of  $X$  and  $Y$  *(4 marks)*
- (iii) (a) Calculate the marginal distribution of  $X$  and  $Y$  *(3 marks)*
  - (b) Find the mean and variance of  $Y$  *(2 marks)*
- (iv) What is the distribution of  $Y$  given that  $X = 2$ ? *(4 marks)*

**End of Question Paper**