



SCHOOL OF MATHEMATICS AND STATISTICS

**Autumn Semester
2011–12**

Foundation Year Mathematics I

1 hour 30 minutes

Attempt all questions. The allocation of marks is shown in brackets.

- 1 Express

$$\frac{x}{x+y} + \frac{x}{x-y} - \frac{1}{(x+y)(x-y)}$$

as a single fraction, simplifying your answer where possible.

(1 mark)

- 2 Simplify

$$\frac{a^2t^3x^4 + 2a^2t^3x^2y^4 + a^2t^3y^8}{ta^2y^8 - ta^2x^4}$$

as much as possible.

(2 marks)

- 3 Simplify

$$\frac{(10)^{\frac{2}{3}} \times (35)^{-\frac{1}{3}} \times \sqrt[3]{14}}{(56)^{-\frac{1}{3}}}$$

as much as possible. Show all your working, which should include using laws of indices.

(2 marks)

- 4 Rationalize the denominator of $\frac{2\sqrt{2}}{1-5\sqrt{2}}$.

(1 mark)

5 (i) Factorize $4x^2 - 7x - 2$. (1 mark)

(ii) Solve the simultaneous equations

$$\begin{cases} 3x^2 - 6y - x = 20 - x^2 \\ y - x + 3 = 0. \end{cases}$$

(4 marks)

6 (i) Complete the square for $x^2 + 6x + 8$. (1 mark)

(ii) Solve $x^2 + 6x + 8 = 0$. (2 marks)

(iii) Draw the graph of $y = x^2 + 6x + 8$ and indicate clearly the minimum point of the curve and where the curve crosses the x - and y -axes. (2 marks)

(iv) Let f be the function with domain $\{x \in \mathbb{R} : -6 < x \leq -1\}$ and rule $f(x) = x^2 + 6x + 8$. Find the range of f . (1 mark)

7 Let $p(x) = x^4 - 7x - 2$, $q(x) = 4x^3 - 5$ and $r(x) = 2x^2 + 2x + 1$.

(i) Expand $p(x) - q(x)r(x)$, collecting like terms together. What is the coefficient of x^3 in $p(x) - q(x)r(x)$? (2 marks)

(ii) Find $r(q(x))$. (2 marks)

8 Express the following fractions as partial fractions. Your answer should include a check.

(i) $\frac{4x - 11}{(x + 4)(2x - 1)}$. (5 marks)

(ii) $\frac{2x^5 - 8x^3 + 5x^2 - 4}{x^2(x^2 - 4)}$. (8 marks)

- 9 Let f be the function with rule $f(x) = 1 + \frac{x}{2-x}$ and domain $\{x \in \mathbb{R} : x \neq 2\}$.
- (i) Find the inverse function f^{-1} . *(3 marks)*
 - (ii) What is the domain of f^{-1} ? *(1 mark)*
 - (iii) What is the range of f^{-1} ? *(1 mark)*

- 10 Sketch the graphs of the following functions between $-\pi$ and π , making sure to mark important points.

- (i) $y = \tan(x)$. *(1 mark)*
- (ii) $y = \tan(x) - 1$. *(2 marks)*
- (iii) $y = -\tan(x)$. *(1 mark)*

- 11 Simplify

$$\ln(e^{x+2} + 2e^{x+1} + e^x) - x. \quad (3 \text{ marks})$$

You should show all your working.

- 12 Solve the following equation for x , giving your answer in terms of $\ln 2$ and $\ln 5$. Your answer must be **exact**, so do not use a calculator.

$$5^{x-1} = \frac{(4e)^x}{10^{x+1}}.$$

(4 marks)

End of Question Paper



SCHOOL OF MATHEMATICS AND STATISTICS

Spring Semester
2011-2012

Mathematics I

3 hours

Attempt all the questions. The allocation of marks is shown in brackets.

- 1 Differentiate the following functions with respect to x :
- (i) $f(x) = 3x^3 + 2 - 3x^{-1}$;
 - (ii) $f(x) = (\sin x) \cdot (\ln x)$;
 - (iii) $f(x) = \frac{e^x}{\cos x}$;
 - (iv) $f(x) = e^{\sin x}$. (8 marks)
- 2
- (i) State the definition of the derivative of a function $f(x)$ (your definition should involve a limit). (3 marks)
 - (ii) Use the method of first principles to differentiate $f(x) = x^{-1}$. (5 marks)
- 3
- (i) Find all stationary points on the curve described by the graph of the function $f(x) = \frac{x^6}{6} - x^4$, and determine their nature. (8 marks)
 - (ii) Use the information from Q3 (i) to sketch the graph of $f(x)$. (4 marks)
- 4
- (i) Find $\frac{dy}{dx}$ when $x(t) = e^{2t}$ and $y(t) = \tan t$. (4 marks)
 - (ii) Find $\frac{dy}{dx}$ when $xy + 3y + x^2 = 0$. (4 marks)
 - (iii) Find $\frac{dy}{dx}$ when $y = 3^x$. (**Hint:** take natural logarithms and use implicit differentiation). (4 marks)

- 5 (i) You are given that the surface area of a sphere of radius r is $4\pi r^2$ and that the radius of the sphere is decreasing at a rate of 1mm per second. Calculate the rate of change of the surface area when the radius of the sphere is 1cm. **(5 marks)**

- (ii) Determine the maximum area of a rectangle with fixed perimeter 8cm. **(5 marks)**

- 6 Find the indefinite integrals in Parts (i) and (ii), and evaluate the definite integrals in Parts (iii) and (iv). You should simplify your answers as much as possible.

(i) $\int 3x^2 - x^{-2} dx;$

(ii) $\int xe^{-x^2} dx.$

(iii) $\int_1^2 \frac{1}{x} dx;$

(iv) $\int_{\ln 2}^{\ln 3} xe^x dx.$ **(10 marks)**

- 7 Differentiate $f(x) = \sin^{-1}(-\sqrt{x^3})$ with respect to x . Hence or otherwise find $\int \sqrt{\frac{x}{1-x^3}} dx.$ **(8 marks)**

- 8 Find the following indefinite integrals:

(i) $\int (\sin(x))^3 dx;$

(ii) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx.$ **(10 marks)**

- 9 Let $f(x) = x^2 - 2x - 3$ and $g(x) = (x + 1)(x^2 - 4)$. By simplifying $\frac{f(x)}{g(x)}$ evaluate $\int_3^4 \frac{f(x)}{g(x)} dx.$ **(10 marks)**

- 10 (i) Determine the area of the region bounded by the lines $x = -1$, $x = 1$ and the graphs of the functions $f(x) = x^3$ and $g(x) = -x^3$. (**Remark:** the region looks a fancy bowtie \bowtie).
- (ii) Using the fact that the semi-circle in the upper half plane centred at $(0, 0)$ of radius r is the graph of the function $f(x) = +\sqrt{r^2 - x^2}$, find the volume of the sphere of radius r . **(12 marks)**

End of Question Paper