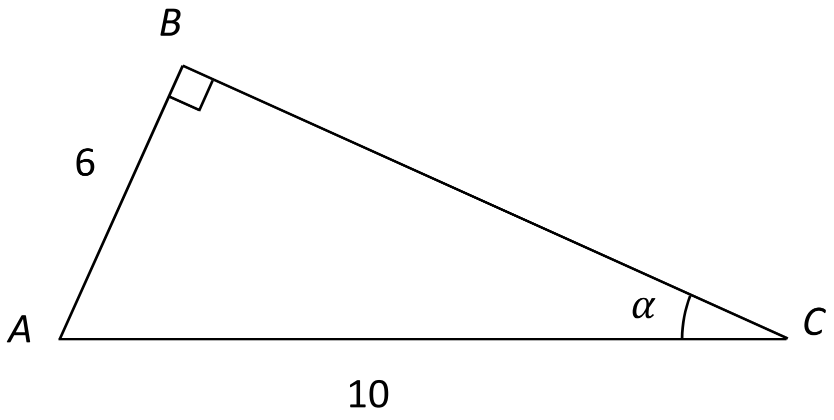
The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Autumn Semester 2011–12**

FOUNDATION YEAR MATHEMATICS II **1 hour 30 minutes**

Attempt all the questions. The allocation of marks is shown in brackets.

1 For the triangle below, calculate $\cos\alpha$ and $\operatorname{cosec}\alpha$.



(3 marks)

2 Draw a detailed sketch of the graph of $y = \tan x$ for $-180^\circ \leq x < 180^\circ$. Find all the values of x in the range $-180^\circ \leq x < 180^\circ$ for which $\tan x = 2$. *(4 marks)*

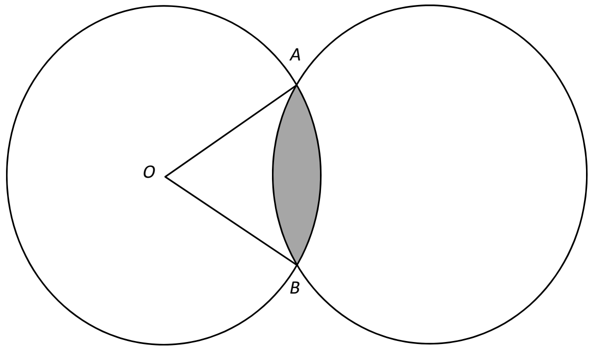
3 (i) Prove that $\sin 45^\circ = \dfrac{1}{\sqrt{2}}$. *(2 marks)*

(ii) Prove that $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$. *(2 marks)*

(iii) For the triangle ABC, using the standard notation, angle B is 120°, angle C is 45° and length c is $\sqrt{30}$. Find the value of length b in surd form. *(3 marks)*

4 (i) Express (a) 135° in radians; (b) $\frac{\pi}{6}$rad in degrees. ***(2 marks)***

(ii) Consider the 2 circles shown below whose circumferences intersect at points A and B, and one circle has centre O. If both circles have a radius of 6 and the length of the perimeter of the combined shape is 20π, find angle AOB and hence calculate the area of the shaded region. ***(5 marks)***



5 Let $A = (1, -5)$ and $B = (4, 7)$.

(i) Find the equation of the line that passes through A and B.

(ii) Find the acute angle between the x-axis and lines perpendicular to AB.

(4 marks)

6 An object is moving at a speed of $v = 15\text{m/s}$ in the direction of $\overrightarrow{OP}$ where $P = (3, \sqrt{15}, -1)$. Find the object's velocity $\mathbf{v}$. ***(2 marks)***

7 Let $ABCD$ be a quadrilateral with $\overrightarrow{AD} = \mathbf{p}$, $\overrightarrow{AB} = \mathbf{q}$ and $\overrightarrow{BC} = \mathbf{r}$. Let E be the midpoint of CD. Show that if $\mathbf{p} + \mathbf{q} = \mathbf{r}$ then $\overrightarrow{EB} = -\mathbf{p}$. ***(4 marks)***

8 Given vectors $\mathbf{a} = (6, 0, 8)$ and $\mathbf{b} = (-4, 4, 2)$, calculate

(i) $0.5\mathbf{a} - \mathbf{b}$,

(ii) $\mathbf{a} \cdot \mathbf{b}$,

(iii) the sum of unit vectors, $\hat{\mathbf{a}} + \hat{\mathbf{b}}$. ***(5 marks)***

9 Let $A = (3,1,1)$, $B = (-1,1,3)$ and $C = (0,2,1)$ be points in 3-dimensions.

(i) Calculate $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form. ***(2 marks)***

(ii) Give the geometric interpretation of the direction cosines of $\overrightarrow{BC}$. ***(2 marks)***

(iii) Calculate the direction cosines of $\overrightarrow{BC}$. ***(2 marks)***

(iv) Calculate $\overrightarrow{BA} \times \overrightarrow{BC}$, and so the area of the triangle ABC. ***(3 marks)***

10 Find the vector equation of the line L_1 which passes through the points $A = (2,0,-9)$ and $B = (3,2,-6)$. Show that L_1 does not intersect the line L_2 with vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + s \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}.$$

(5 marks)

End of Question Paper

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SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2011–2012**

FOUNDATION YEAR MATHEMATICS II **3 hours**

Attempt all the questions. The allocation of marks is shown in brackets.

1 (i) A ship starts at point A and travels North-East for 6km to reach point B, then changes direction and travels North for 8km to point C. When the ship reaches point C, the captain notices that fuel is running low and the ship can only manage a further 13km. The options are

(a) travel to an island which lies North-West of A and West of C, or

(b) travel back to point A by the direct route.

Which option should the captain choose? (You must justify your answer for marks.) ***(6 marks)***

(ii) Three circular discs, each of radius r, touch each other. Given that the area enclosed between them is $5cm^2$, find the radius of the circles to 2 decimal places. ***(6 marks)***

(iii) Two particles are travelling in space. At time t (in seconds), particle A has position vector given by

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

whereas particle B has position vector

$$\mathbf{r} = \begin{pmatrix} -39 \\ -99 \\ -7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -\frac{3}{2} \end{pmatrix}.$$

Show that the paths of the particles do cross but that they do not collide, since they arrive at this point of intersection precisely 1 minute apart. ***(8 marks)***

2 (i) Sketch the graphs of the following functions for $-2\pi \leq x \leq 2\pi$. Your sketches should have all important points, crossings and asymptotes calculated and labelled.

(a) $y = \cos\left(x + \frac{\pi}{4}\right)$,

(b) $y = \tan(x/2)$,

(c) $y = 2\cos(x) - 1$. ***(9 marks)***

(ii) The water depth, d metres, in a harbour is given by the expression

$$d = 3\sin t + 4\cos t + 6,$$

where t denotes time, measured in hours. Find the maximum water depth and give the values of t in the range $0 \leq t \leq 12$ at which this depth occurs. ***(7 marks)***

(iii) By using an appropriate addition formula, and the fact that $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$, find the precise values of $\sin\left(\frac{\pi}{12}\right)$ and $\cos\left(\frac{\pi}{12}\right)$. Hence evaluate $\tan\left(\frac{\pi}{12}\right)$ precisely. (You must show your workings for marks.) ***(4 marks)***

3 (i) Find the sum of each of the series below, showing your workings.

(a) $1 + \dfrac{5}{4} + \dfrac{3}{2} + \dfrac{7}{4} + 2 + \ldots + 12$;

(b) $3 + 1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \ldots$;

(c) $\displaystyle\sum_{m=0}^{\infty} \left(\frac{1}{2}\right)^m$. ***(9 marks)***

(ii) Express the recurring decimal $0.621621\dot{6}2\dot{1}\ldots$ as a fraction in its lowest terms. ***(3 marks)***

(iii) (a) How many different ways are there of choosing 11 people to form a team from a squad of 23 players?

(b) How many different ways are there of allocating 11 numbered shirts to a squad of 23 players? (There is one shirt with '1' on the back, one with '2' on the back, etc.) ***(4 marks)***

(iv) Write down the expansion of $(1 + x)^n$, where n is a positive integer. By substituting an appropriate value for x, show that

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} = 2^n.$$

(4 marks)

4 (i) Let $z_1 = 1 + i$ and $z_2 = 2 - i$. Calculate

(a) $z_1 z_2$;

(b) $\frac{z_1}{z_2}$;

(c) $|z_2|$;

(d) the principal argument of z_2;

(e) the conjugate $\overline{z_1}$. ***(7 marks)***

(ii) Find all four solutions, real and complex, of the equation $z^4 - 9 = 0$. ***(3 marks)***

(iii) Consider the curve in the (x, y)-plane given by $x^2 + y^2 - 6x + 2y + 6 = 0$.

(a) Give a thorough description of the shape formed by this curve and draw a graph to illustrate your answer. ***(4 marks)***

(b) One point which lies on the curve is $P = (3 - \sqrt{3},\ 0)$. Find the equation of the tangent to the curve at P. ***(4 marks)***

(iv) The parametric equations $x = 2t - 1$ and $y = 3 - t$ (where $-\infty < t < \infty$) represent a straight line in the (x, y)-plane. Find the gradient of this line. ***(2 marks)***

5 (i) Use the iterative formula

$$x_{r+1} = \sqrt{\frac{3}{x_r}}$$

with $x_1 = 1.5$ to find $x_2,\ x_3$ and x_4 correct to 3 significant figures. Find, in its simplest form, the equation that is solved by this formula and hence an exact expression for the solution that was approximated by x_4. ***(5 marks)***

(ii) Evaluate

$$\int_0^2 (x^3 - 5x^2 + 6x)\ dx$$

working to three decimal places (and showing your workings)

(a) by ordinary integration;

(b) using the trapezium rule with four strips;

(c) by Simpson's rule with four strips.

(10 marks)

(iii) Find the general solution to the differential equation $\frac{dy}{dx} = 2y$ (where $y > 0$) making the method clear. Hence find the particular solution for which $y = 1$ when $x = \frac{1}{2}\ln 2$. ***(5 marks)***

End of Question Paper