Data provided: Formula Sheet

MAS243

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2011–2012**

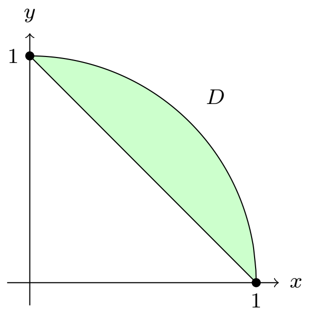
Mathematics IV (Electrical) 2 **hours**

Attempt all FOUR questions. Each question is worth 25 marks.

1 (i) The function $f(x,y) = (2+\cos(x))(2+\sin(y))$ has four critical points with $0 \le x, y < 2\pi$. Find and classify these critical points, and determine the maximum and minimum values of f. ***(13 marks)***

(ii) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $x^2+4xy+4y^2$ on the circle of radius $\sqrt{5}$ centred at the origin. ***(12 marks)***

2 Let D be the following region:



(The outer part of the boundary is part of the circle of radius one centred at the origin, and the inner part is a straight line.)

(a) Find appropriate limits to express the integral $I = \iint_D x^2\,dA$ as a double integral in two different orders:

$$I = \int_{x=\cdots}^{\cdots}\int_{y=\cdots}^{\cdots} x^2\,dy\,dx = \int_{y=\cdots}^{\cdots}\int_{x=\cdots}^{\cdots} x^2\,dx\,dy.$$ ***(8 marks)***

(b) Use a substitution to show that $\displaystyle\int_{t=0}^{1} t^2\sqrt{1-t^2}\,dt = \pi/16.$ ***(13 marks)***

(c) Use the first expression from (a) together with (b) to evaluate I. ***(4 marks)***

3 Put $\mathbf{u} = (\cos(z),\ \sin(z),\ \exp(z))$ and $\mathbf{v} = (-\sin(z),\ \cos(z),\ \exp(-z))$.

(i) Simplify the following:

$$\begin{aligned} \mathbf{u}.\mathbf{v} \\ \mathbf{u}.\,\mathrm{curl}(\mathbf{u}) \\ \mathrm{curl}(\mathbf{u}) \times \mathrm{curl}(\mathbf{v}) \\ \mathrm{div}(\mathbf{u})\,\mathrm{div}(\mathbf{v}) \\ \mathbf{u} - \mathrm{curl}(\mathrm{curl}(\mathbf{u})) \\ \mathrm{div}(\mathrm{curl}(\mathrm{curl}(\mathrm{curl}(\mathbf{v})))) \end{aligned}$$

(18 marks)

(ii) Is there a function f such that $\mathbf{u} = \mathrm{grad}(f)$? Justify your answer. ***(2 marks)***

(iii) Find a function g such that $\mathbf{u} = \mathrm{grad}(g) + \mathrm{curl}(\mathrm{curl}(\mathbf{u}))$ ***(5 marks)***

.

4 (a) Let C be the circle of radius one centred at $(1, 0)$, and let $\mathbf{u}$ be the vector field $(x^2 - 2x, x + xy)$. Evaluate $\int_C \mathbf{u}.d\mathbf{r}$. ***(6 marks)***

(b) Let $\mathbf{v}$ be the vector field (z, y, x), and let S be the sphere of radius 2 centred at the origin. Evaluate $\iint_S \mathbf{v}.d\mathbf{A}$ directly (without using the Divergence Theorem).

(14 marks)

(c) Evaluate $\iiint_E xyz\,dV$, where E is the solid region given by $x, y, z \geq 0$ with $y \leq 1$ and $x + z \leq 1$. ***(5 marks)***

End of Question Paper

Formula Sheet for MAS243
Trigonometry

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$a \cos\theta + b \sin\theta = R \cos(\theta - \alpha) \text{ where } R = \sqrt{a^2 + b^2} \text{ and } \cos\alpha = \frac{a}{R},\ \sin\alpha = \frac{b}{R}$$

$$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$$

$$\cos^3\theta = \frac{1}{4}(3\cos\theta + \cos 3\theta)$$

$$\cos^4\theta = \frac{1}{8}(3 + 4\cos 2\theta + \cos 4\theta)$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$$

$$\sin^4\theta = \frac{1}{8}(3 - 4\cos 2\theta + \cos 4\theta)$$

End of Question Paper