The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2011–2012**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5, only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below.

(a) The following extract is translated from the Babylonian clay tablet **BM 13901**.

I add up the area and side of my square: 0;45. Write down 1 the coefficient, and halve it: 0;30. Multiply 0;30 and 0;30: 0;15. Add it to 0;45: 1. This is the square of 1. From 1 subtract 0;30, which was multiplied by itself. The side of the square is 0;30.

What does **BM 13901** signify? Write a modern account of the problem. For what purpose do you think that it was devised? Give reasons for your response. ***(7 marks)***

(b) Describe the *three classical problems of antiquity.* For each, name a Greek who studied it, indicating his contribution. What roles did Pierre Wantzel and Ferdinand Lindemann play in the resolution of the problems? ***(7 marks)***

(c) For each of Robert Recorde, Thomas Harriot and John Napier, state the language in which he wrote his *mathematical* works and name a frequently used item of notation that he introduced. Which *two* of these men published *mathematical* works that had instant popular success? Justify your answer. ***(7 marks)***

(d) Let the tangent T to the curve $y = f(x)$ at the point $(a, f(a))$ meet the x-axis at the point $(a - t, 0)$, where $t > 0$. How can T be constructed once t is known? Obtain the *adequality*

$$\frac{f(a)}{t} \sim \frac{f(a+E)}{t+E} \ \ (\textit{small } E > 0).$$ ***(3 marks)***

How would Fermat show that $t = 2a$, when $y = \sqrt{x}$ and $a > 0$? Deduce the *gradient* of T in this particular case. ***(4 marks)***

2 **(a)** What type of mathematical documents are the *Moscow* and *Rhind Papyri*? In what script are they written? How many problems does each contain? ***(4 marks)***

(b) Write accounts of **Problem 48** on the *Rhind Papyrus* and **Problem 14** on the *Moscow Papyrus*, mentioning points of special interest. ***(12 marks)***

3 Which books of Euclid's *Elements* are devoted to *number theory*? Is the positioning of these books within the work as a whole crucial or could they be moved *en bloc* elsewhere, say to the beginning or to the end? Name *two* topics in early Greek *number theory* that are *not* mentioned in the *Elements*. ***(4 marks)***

The three propositions stated below all occur in *one* book of the *Elements*. Which book is this and how many propositions does it contain? Comment on the placement of the last two propositions within that book. ***(2 marks)***

Proposition 20 *Prime numbers are more than any assigned multitude of prime numbers.*

Proposition 35 *If numbers are in continued proportion, and there be subtracted from the second and the last of these a number equal to the first, then the excess of the second to the first equals the excess of the last to all those before it.*

Proposition 36 *If numbers, beginning with a unit, are set out in double proportion until their sum becomes prime, then this multiplied into the last is perfect.*

(a) Explain *carefully* the wording of **Proposition 20**. How is the result stated in modern texts? Why do you think that Euclid chose the particular way that *he* did? ***(3 marks)***

(b) Use **Proposition 35** to show that, if a and r numbers $(a, r > 0,\ r \neq 1)$, then

$$a + ar + ar^2 + \cdots + ar^{n-1} = a\frac{r^n - 1}{r - 1}.$$ ***(3 marks)***

Deduce that, if $2^n - 1$ is *prime*, then $2^{n-1}(2^n - 1)$ is *perfect*. ***(1 mark)***

(c) The ancient Greeks knew of *four* perfect numbers. How many of these are mentioned in the *Elements*? Use **Proposition 36** to find *four* perfect numbers. ***(3 marks)***

4 Give an account of the problem-solving competition that took place in Venice on the 22 nd February 1535. Indicate its role in the history of the cubic equation. ***(6 marks)***

Problem $\boldsymbol{\alpha}$ and verse $\boldsymbol{\beta}$ below, taken from the same work of 1546, have *direct* relevance to the competition. Identify the author of the work and explain the relevance. ***(3 marks)***

$\boldsymbol{\alpha}$ *A tree 12 units high is cut in two. The height of the tree left standing is the cube root of the length cut away. What is the height of the tree left standing?*

$\boldsymbol{\beta}$ *When the cube and the things together*
Are equal to some discrete number
Find two numbers differing in this one.
Then you will keep this as a habit
That their product should always be equal
Exactly to the cube of a third of the things.
The remainder then as general rule
Of their cube roots subtracted
Will be equal to your principal thing.

Write $\boldsymbol{\alpha}$ as a cubic equation in the height x of the tree left standing. Use $\boldsymbol{\beta}$ to show that

$$x = \sqrt[3]{6 + \sqrt{973/27}} - \sqrt[3]{-6 + \sqrt{973/27}}\,.$$ ***(7 marks)***

5 Archimedes' use of the *method of exhaustion*, together with his *balancing method* for finding areas and volumes of curved figures, marks the pinnacle of Greek mathematics, but his ideas were not taken up by others. Suggest reasons for this. Name *one* advantage and *two* disadvantages of the method of exhaustion. Why did Archimedes *need* to employ *both* methods of demonstration? ***(6 marks)***

State Archimedes' result on the area of a parabolic segment S. Mention *two* works in which he derived the result, one by *exhaustion*, the other by *balancing*. Describe the first *two* terms in the sequence $P_0, P_1, \ldots$ of polygons that he used to *exhaust* S. Express the area of P_1 in terms of that of P_0. ***(5 marks)***

Consider the parabola $x = y^2$ relative to rectangular x- and y- coordinate axes. Let

$$O = (0,0), \quad U = (h^2, -h), \quad V = (0, -h), \quad W = (0, h), \quad X = (h^2, h), \ \text{ where } \ h > 0.$$

Let C be the cylinder of radius h and height h^2 formed by rotating the rectangle $UVWX$ about the x- axis, and let P be the paraboloid of revolution formed by rotating the parabolic segment UOX about the x- axis. Show that the vertical circular slice of P at x $(0 \le x \le h)$, when placed with centre at $(-h^2, 0)$, balances the vertical slice of C at x about fulcrum O. Deduce that the volume of P is *half* that of C. ***(5 marks)***

End of Question Paper