The University Of Sheffield.

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Neaves Tables
Graph Paper

SCHOOL OF MATHEMATICS AND STATISTICS — MAS6011

Session 2011-2012 — **3 Hours**

Dependent Data

RESTRICTED OPEN BOOK EXAMINATION.
Candidates may bring to the examination lecture notes and associated lecture material (but no textbooks) plus a calculator that conforms to University regulations.
*All answers will be marked but credit will be given for only the best **FIVE** answers.*
All questions carry equal marks. Total marks 100.

Registration number from U-Card (9 digits) – to be completed by student

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1 A study was made of samples of Romano-British glass from two known production centres at Chester and Doncaster. Particular attention was given to the quantities of potassium (K) and lithium (Li) since this is known to depend upon the local materials and so can be distinctive of the production centre. The 31 samples from Chester gave mean quantities for K and Li of 8.4 and 19.5 respectively with sample variances 1.84 and 2.23 respectively and covariance 0.24 (measurements are in parts per thousand). The measurements of the 41 samples from Doncaster gave mean results of 9.6 and 18.3 with sample variances 1.64 and 2.00 and sample covariance 0.18.

(a) Calculate Fisher's linear discriminant function for classifying a piece of glass with quantities (x_1, x_2) of K and Li as coming from Chester.

(7 marks)

(b) In order to simplify the calculations it is decided to use a rule of classifying a piece of glass as coming from Chester if the difference in content of lithium and potassium is greater than 8 parts per thousand. Stating any distributional assumptions you make, estimate the probability of misclassifying a randomly selected Chester sample as being produced in Doncaster when using this rule.

(4 marks)

(c) With the aid of a rough diagram and without necessarily doing any detailed calculations, is the probability calculated in part (b) more or less than that using Fisher's linear discriminant function?

(2 marks)

(d) Estimate the probability of misclassifying a randomly selected Doncaster sample as being produced in Chester when using the simplified rule in part (b).

(3 marks)

(e) A piece of glass found in Buxton has potassium and lithium contents of 9.0 and 18.9 parts per thousand respectively. How would you classify the piece using each of the two rules specified in parts (a) and (b) above?

(4 marks)

2 As part of an investigation into determining possible locations of early hominids (i.e. ancient ape-like ancestors of humans) outside Africa, data were collated giving the numbers of fossil species of non-hominid fauna in various categories found at 139 different sites in Africa and Asia. It is thought that the profile of non-hominid types of fauna is related to whether or not hominids are attracted to the site. These sites included 21 sites (which are all African) where early hominid fossil remains have been found; no hominids have been found in the other 118 sites, some of which are in Asia and the others in Africa. The five categories recorded were Herbivores, Herbivore/Root-Eaters, Carnivores, Carnivore/Scavengers and Omnivores. Given below is a record (edited in places) of various preliminary analyses of these data using **R**. (NB: Herbivores eat vegetation only, Carnivores eat primarily meat and Omnivores eat both.)

(a) The principal component analysis has been performed using the correlation matrix. Would you recommend instead using the covariance matrix? Justify your recommendation.

(2 marks)

(b) Basing your judgement on some informal graphical technique (which should be given), how many principal components would you recommend retaining for further exploratory analyses?

(3 marks)

(c) What features of the sites do the most important principal components reflect?

(5 marks)

(d) What characteristics of the sites (in terms of the categories of fossil species found at them) seem to be typical of the majority of the hominid sites?

(5 marks)

(e) Two further sites in Asia are under consideration for further intensive excavation in the hope of identifying early hominid fossils but resources are only sufficient for a single expedition to one of the sites. The numbers (respectively) of Herbivores, Herbivore/Root-Eaters, Carnivores, Carnivore/Scavengers and Omnivores recorded at Site A are 20, 2, 0, 1 and 0. At Site B they were 9, 0, 4, 1 and 1. Upon which site would you recommend concentrating the available resources? Justify your answer.

(5 marks)

```
> hominids[1:4,]
  Herbivores Herb.Root Carnivores Carn.Scavangers Omnivores Hominid
1         43        10          4               4         0       0
2         21         2          4               2         1       0
3          5         0          0               0         0       0
4          6         1          1               1         1       0
> hominids[117:121,]
    Herbivores Herb.Root Carnivores Carn.Scavangers Omnivores Hominid
117          8         0          2               1         0       0
118         12         0          4               1         0       0
119          0         0          1               0         0       1
120          8         1          0               0         0       1
121         25         3          1               0         0       1
```

Question 2 continued on next page

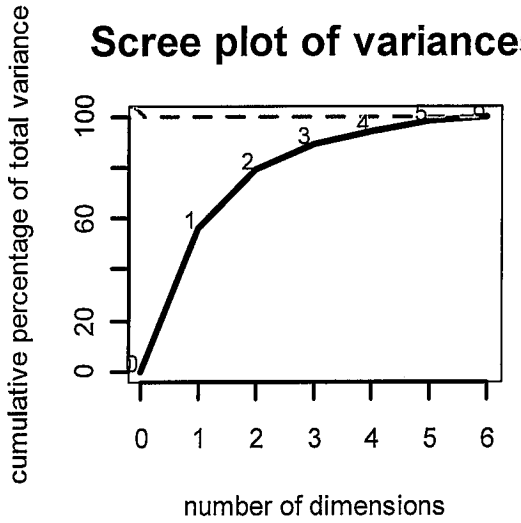
Question 2 continued

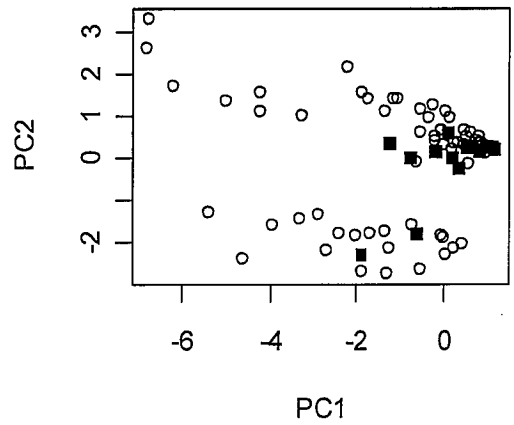
```
          ***  Summary Statistics for data in:  hominids ***
Hominid:1
         herbivores herb.root carnivores carn.scavangers omnivores
    Mean:       4.43     0.364       1.25          0.364     0.195
 Total N:     118.00   118.000     118.00        118.000   118.000
Std Dev.:       8.11     1.152       2.30          0.724     0.398
-----------------------------------------------
Hominid:2
         herbivores herb.root carnivores carn.scavangers omnivores
    Mean:       18.8      3.00       2.71          0.952    0.0952
 Total N:       21.0     21.00      21.00         21.000   21.0000
Std Dev.:       13.5      2.35       3.23          1.284    0.3008
```

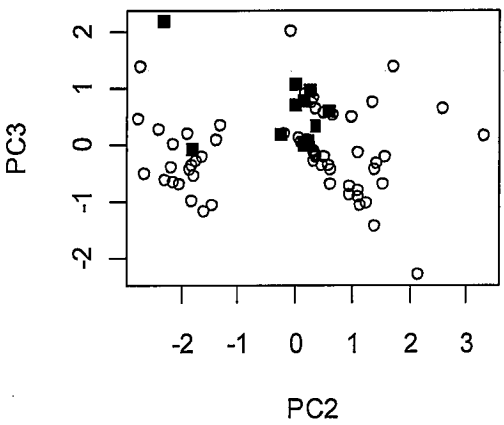
```
>attach(hominids)
> hom.pc<-prcomp(hominids[-6],scale=T)
> options(digits=2)
> summary(hom.pc)
Importance of components:
                          PC1   PC2    PC3    PC4   PC5
Standard deviation     1.78 1.074 0.5647 0.4977 0.361
Proportion of Variance 0.63 0.231 0.0638 0.0495 0.026
Cumulative Proportion  0.63 0.861 0.9245 0.9740 1.000
> options(digits=1)
> hom.pc$rotation
                  PC1   PC2   PC3    PC4   PC5
Herbivores       -0.5  0.23 -0.36 -0.003 -0.74
Herb.Root        -0.5  0.42 -0.38 -0.225  0.64
Carnivores       -0.5 -0.31  0.08  0.794  0.19
Carn.Scavangers -0.5  0.03  0.79 -0.364 -0.03
Omnivores        -0.2 -0.82 -0.31 -0.432  0.05
>  par(mfrow=c(2,2))
>  screeplot(hominids,cor=T)
>  plot(hom.pc$x[,1:2],type ='n')
>  points(hom.pc$x[1:118,1:2],pch=1)
>  points(hom.pc$x[119:139,1:2],pch=15)
>  plot(hom.pc$x[,2:3],type ='n')
>  points(hom.pc$x[1:118,2:3],pch=1)
>  points(hom.pc$x[119:139,2:3],pch=15)
>  plot(hom.pc$x[,3:4],type ='n')
>  points(hom.pc$x[1:118,3:4],pch=1)
>  points(hom.pc$x[119:139,3:4],pch=15)
```

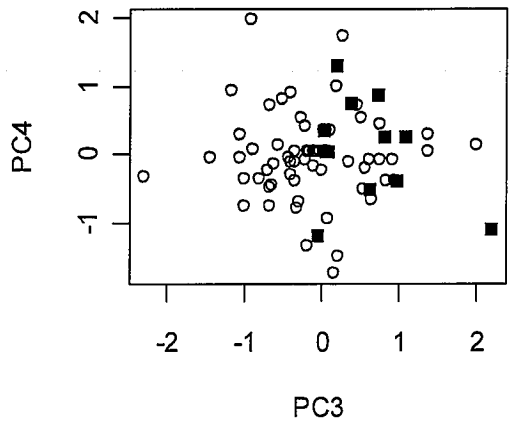
Question 2 continued on next page

Question 2 continued









3 Johnson & Wichern (2002) provide data giving the speeds in metres per second attained by men and women achieving the national record times (as they stood at the 1984 Los Angeles Olympics) in races over distances from 100 metres to 5000 metres in each of 55 different countries. These distances were, in metres, 100, 200, 400, 800, 1500 and either 3000 or 5000 (for women and men) and the marathon. A record of some multivariate analyses of these data is given below, where these various races are referred to as and m150m for the 1500 metres. The 3000m and 5000m are both coded as m5000m. Seven of the 55 countries are former 'Eastern block countries: China, Czechoslovakia, East Germany, Hungary, North Korea, Poland and Romania. Men and women are coded in the variable `gender` as 1 and 4 in the former Eastern block countries; 0 and 3 in the in the other 48 countries. Given below is a record (edited in places) of various preliminary analyses of these data using **R**.

(a) What evidence is provided by the various analyses of data that the record times differ systematically between the four groups?

(5 marks)

(b) Justifying your answer, what linear combination of the four measurements shows the greatest differences between the four groups?

(3 marks)

(c) What is the statistical significance of the difference measured by this combination of variables?

(2 marks)

(d) Overall, what is the major difference in record running speeds between men and women?

(3 marks)

(e) In terms of running speeds what distinguishes between the record running speeds of women from the Eastern block and other countries?

(3 marks)

(f) How strong is the evidence that there is a difference between record running speeds of women from the Eastern block and other countries?

(2 marks)

(g) How strong is the evidence that there is a difference between record running speeds of men from the Eastern block and other countries?

(2 marks)

Question 3 continued on next page

Question 3 continued

```
> library(MASS)
> attach(arrowmin)

> olymprecs[53:58,]
   gender m100m m200m m400m m800m m1500m m5000m mmarathon  country
53      0  10.1  10.1   9.1   7.7    7.1    6.3       5.5      usa
54      1   9.9  10.0   9.0   7.6    7.0    6.3       5.4     ussr
55      0   9.2   9.1   8.2   6.6    5.9    5.1       4.3   wsamoa
56      3   8.6   8.7   7.3   6.2    5.6    5.1       3.9 argentin
57      3   8.9   8.9   7.8   6.7    6.1    5.5       4.6 australi
58      3   8.7   8.7   7.9   6.7    5.9    5.4       4.4  austria
> ly.pc<-prcomp(olymprecs[,-c(1,9)],scale=T)
> options(digits=2)
> summary(oly.pc)
Importance of components:
                          PC1    PC2    PC3     PC4     PC5    PC6     PC7
Standard deviation     2.553 0.5281 0.2982 0.23982 0.16059 0.1322 0.11370
Proportion of Variance 0.931 0.0398 0.0127 0.00822 0.00368 0.0025 0.00185
Cumulative Proportion  0.931 0.9710 0.9838 0.99197 0.99566 0.9981 1.00000
> oly.pc$rotation
             PC1    PC2    PC3    PC4    PC5    PC6    PC7
m100m     -0.37 -0.458 -0.382  0.267  0.254 -0.594 -0.123
m200m     -0.38 -0.417 -0.326  0.041 -0.071  0.698  0.287
m400m     -0.38 -0.272  0.351 -0.277 -0.651 -0.098 -0.384
m800m     -0.38 -0.043  0.566 -0.205  0.305 -0.175  0.604
m1500m    -0.38  0.209  0.282  0.206  0.495  0.325 -0.579
m5000m    -0.37  0.520 -0.049  0.603 -0.406 -0.085  0.231
mmarathon -0.37  0.476 -0.472 -0.634  0.063 -0.081 -0.029

> oly.lda<-lda(gender~as.matrix(olymprecs[,-
c(1,9)],prior=c(.25,.25,.25,.25)))
> oly.lda
Call:
lda(gender ~ as.matrix(olymprecs[, -c(1, 9)], prior = c(0.25,
    0.25, 0.25, 0.25)))

Prior probabilities of groups:
    0     1     3     4
0.436 0.064 0.436 0.064
Group means
```

	m100m	m200m	m400m	m800m	m1500m	m5000m	marathon
0	9.5	9.5	8.6	7.4	6.8	6.0	5.1
1	9.7	9.7	8.8	7.5	6.9	6.2	5.3
3	8.6	8.4	7.4	6.4	5.8	5.3	4.1
4	8.8	8.8	8.0	6.9	6.2	5.7	4.4

Question 3 continued on next page

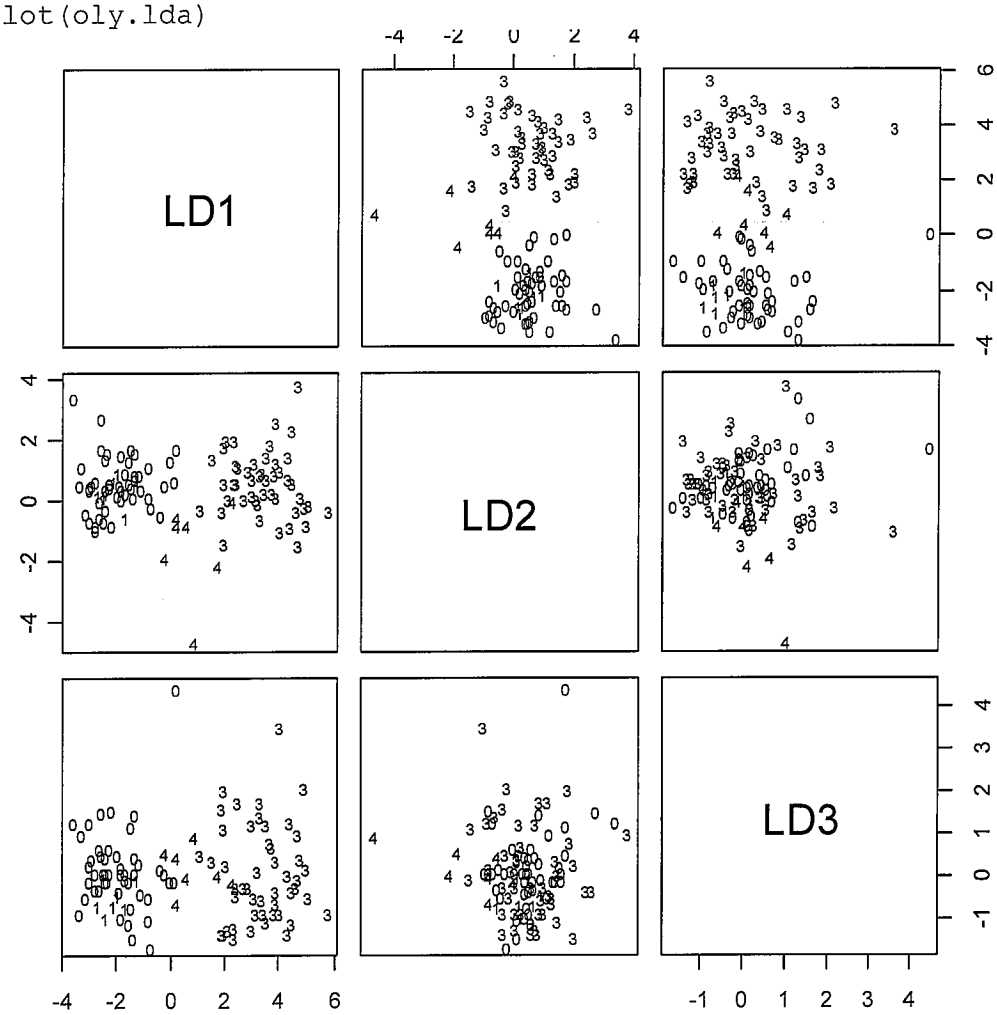
Question 3 continued

```
Coefficients of linear discriminants:

                LD1    LD2   LD3
m100m         0.902  2.08 -4.16
m200m        -0.294  2.47  2.88
m400m        -2.533 -5.64 -1.90
m800m        -3.248  0.18  4.49
m1500m       -2.827  2.18  1.91
m5000m        4.628 -4.80 -4.29
mmarathon     0.003  2.99 -0.39


Proportion of trace:
   LD1    LD2    LD3
0.9563 0.0399 0.0038
>#  find predicted classifications
#  without cross-validsation/jackknifing

> olydata<-data.frame(olymprecs[,-c(1,9)])
>
> oly.pred<-predict(oly.lda,olydata)
>
gender  0  1  3  4
     0 48  0  0  0
     1  7  0  0  0
     3  0  0 46  2
     4  0  0  1  6

#  find predicted classifications
#  with cross-validsation/jackknifing
>
> oly.predCV<-predict(oly.lda,olydata,CV=T)
>
> table(gender,oly.predCV$class)

gender  0  1  3  4
     0 48  0  0  0
     1  7  0  0  0
     3  0  0 46  2
     4  0  0  1  6
>
```

Question 3 continued on next page

Question 3 continued

```
# MANOVA : East vs west
> oly.manova<-manova(as.matrix(olymprecs[,-c(1,9)])~gender)
> summary(oly.manova)
           Df Pillai approx F num Df den Df Pr(>F)
gender      1  0.758     45.7      7    102 <2e-16 ***
>
# MANOVA on men: East vs west
> oly.men.manova<-manova(as.matrix(
+ olymprecs[1:55,-c(1,9)])~gender[1:55])
>
> summary(oly.men.manova)
             Df Pillai approx F num Df den Df Pr(>F)
gender[1:55]  1 0.0721    0.522      7     47   0.81
Residuals    53
# MANOVA on women: East vs west
> oly.women.manova<-manova(as.matrix
+ (olymprecs[56:110,-c(1,9)])~gender[56:110])
> summary(oly.women.manova)
               Df Pillai approx F num Df den Df Pr(>F)
gender[56:110]  1  0.537      7.8      7     47  3e-06 ***
> plot(oly.lda)
```



4 (i) The figure below shows the quarterly averages of the Euro/£ and US$/£ exchange rates between 2001 and 2005 (source: Bank of England). Also plotted is the price of gold relative to its price in the first quarter of 2001. Describe the three time series and their relationship, using suitable technical terms and adding approximate quantification where appropriate. Detailed numerical comparisons of the series are not required. *(4 marks)*

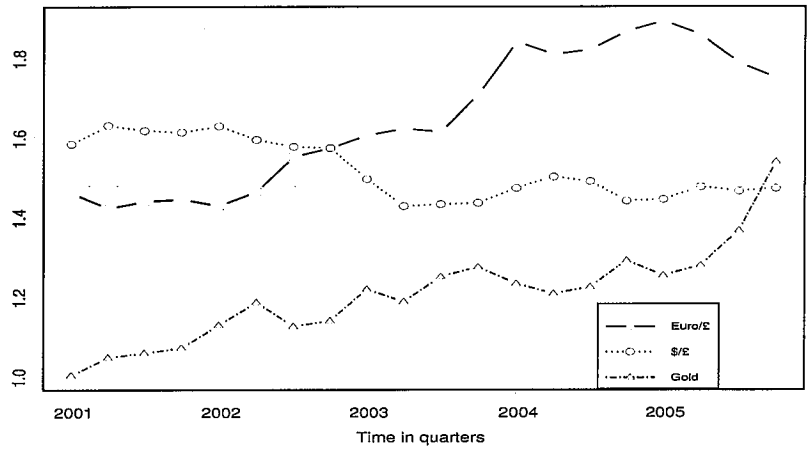


Figure 1: Dollar/Pound, Euro/Pound and Gold Index, Jan 2001 - Dec 2005

(ii) The following table shows the sample acf r_h and pacf $a_h^{(h)}$ values of the Dollar/pound series where the lag $h = 1, 2, 3, \ldots$ is in quarters of a year.

Lag h	1	2	3	4	5	6	7	8
r_h	0.9123	*	0.6287	0.4604	0.2703	0.0898	-0.0653	-0.2012
$a_h^{(h)}$	*	-0.3586	-0.0071	-0.2779	-0.1745	-0.0563	-0.0252	-0.0706

Calculate the two missing values. Without making any further calculations, describe briefly how using the above table, you could investigate whether the time series is stationary or not. *(7 marks)*

(iii) Investigate and compare possible models for the series given the values in the table in (ii) and your calculations for the missing ones. What further evidence is desirable for a more complete identification of a suitable model? *(9 marks)*

5 Consider the time series model

$$X_t = \frac{1}{3}X_{t-1} + \epsilon_t + \frac{1}{2}\epsilon_{t-1} - \frac{1}{4}\epsilon_{t-2},$$

where ϵ_t is white noise with variance 4.

(i) Explain why in this model X_t has zero mean. *(2 marks)*

(ii) Show that this model is invertible. *(3 marks)*

(iii) If $Var(X_t) = 9$, calculate the autocorrelation function (ACF) of X_t. *(10 marks)*

(iv) Find a state space representation for the model for X_t. Write down the observation and evolution equations and state the distribution of the observation and evolution innovations. *(5 marks)*

6 (i) Consider the AR(1) time series model

$$X_t = \alpha X_{t-1} + \epsilon_t,$$

where ϵ_t is a white noise sequence with variance σ^2.

(a) Derive the formula

$$X_{n+k} = \alpha^k X_n + \sum_{i=0}^{k-1} \alpha^i \epsilon_{n-i}$$

for some positive integers n and k. *(2 marks)*

(b) Using (a) or otherwise, derive the k-step ahead forecast mean and variance of X_{n+k}, based on observed time series data $X_1 = x_1, \ldots, X_n = x_n$. *(7 marks)*

(c) If ϵ_t is normally distributed, write down a 95% prediction interval for X_{n+k}. *(1 mark)*

(ii) Consider the AR(2) time series model

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \epsilon_t,$$

where ϵ_t is as above a white noise sequence with variance σ^2.

(a) Write down X_t in state-space form. *(1 mark)*

(b) Using the state-space form in (a), derive the 2-step ahead forecast mean and variance of X_{n+2}, based on observed time series data $X_1 = x_1, \ldots, X_n = x_n$. *(7 marks)*

(c) Hence show that the 2-step forecast variance of X_{n+2} is the same as that of an AR(1) model defined by $X_t = \alpha_1 X_{t-1} + \epsilon_t$. *(2 marks)*

End of Question Paper