



SCHOOL OF MATHEMATICS AND STATISTICS

Spring Semester  
2012-2013

FOUNDATION YEAR MATHEMATICS 1

3 hours

Attempt *all* questions. The allocation of marks is shown in brackets. Total marks: 100.

1 Differentiate  $f(x) = 5x^2 - 7$  from first principles. (4 marks)

2 Differentiate the following with respect to  $x$ .

(i)  $y = 4x^{10} - 7\sqrt[3]{x} + \frac{4}{x^{\frac{5}{6}}} + 9;$

(ii)  $y = (x^3 + 4) \ln x;$

(iii)  $y = \cos\left(2x - \frac{\pi}{7}\right);$

(iv)  $y = \frac{e^x}{\sin^{-1} x}.$  (10 marks)

You are expected to simplify each of your answers as much as possible.

3 Find the following indefinite integrals.

(i)  $\int \left(4x^8 + \frac{7}{x^8} - 5\sqrt[4]{x} + \pi\right) dx;$

(ii)  $\int (4x - 1) \sin(4x^2 - 2x + 1) dx;$

(iii)  $\int (1 - 8x)e^x dx.$  (9 marks)

**4** Evaluate the following definite integrals.

(i)  $\int_2^4 x^3 dx;$

(ii)  $\int_{-\pi}^{\pi} \cos\left(\frac{1}{2}x - \pi\right) dx. \quad (5 \text{ marks})$

**5** Using parametric differentiation, find  $\frac{dy}{dx}$  in terms of  $t$  when  $y = \cos^{-1} t$  and  $x = \sqrt{1 - t^2}$ . *(3 marks)*

**6** Let  $y = 4^{(x^2)}$ . Find  $\frac{dy}{dx}$ . *(3 marks)*

**7** (i) Express

$$\frac{1}{1 - x^4}$$

as partial fractions. Your answer should include a check. *(11 marks)*

(ii) Find

$$\int \frac{1}{1 - x^4} dx. \quad (3 \text{ marks})$$

**8** (i) Find the stationary points of the curve

$$y = f(x) = 9x^5 - 5x^9$$

and determine their nature. *(9 marks)*

(ii) Sketch the graph of  $y = f(x) = 9x^5 - 5x^9$ . *(8 marks)*

**9** For  $\frac{\pi}{6} \leq x \leq \pi$ , find the area enclosed by the curve  $y = \sin x$  and the straight line given by the points  $P = \left(\frac{\pi}{6}, \frac{1}{2}\right)$  and  $Q = (\pi, 0)$ . *(7 marks)*

**10** Find

$$\int \left( \frac{18x^9 + 25x^8 + 22x^7 + 19x^6 + 16x^5 + 5x^4 + 1}{2x + 1} \right) dx.$$

*(8 marks)*

- 11** Find the equations of the tangent and the normal to the curve  $y = f(x) = e^{2x+1}$  at the point  $P = (1, e^3)$ . **(7 marks)**

- 12** Find the derivative of

$$y = \ln \left( \frac{(x^4 \sin x^2)^3 (x+1)^7}{e^x \ln x} \right). \quad (8 \text{ marks})$$

- 13** Find the volume swept out when the area under  $y = x + 1$  enclosed by  $y = 0$ ,  $x = 0$  and  $x = 2$  is rotated about the  $x$ -axis through  $2\pi$  radians. **(5 marks)**

**End of Question Paper**