Data provided: Formula Sheet

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2012–2013**

MAS156 Mathematics (Electrical and Aerospace) **2 hours 30 minutes**

Attempt ***ALL*** *questions.*
Each question in Section A carries 2 or 3 marks and each question in Section B carries 8 marks.

Section A

A1 Simplify the following expression as much as possible: $e^{-\ln y}e^{\ln(y^2)}$. ***(2 marks)***

A2 Give an example of each of the following: an even function, an odd function and a function which is neither even nor odd. ***(3 marks)***

A3 Find $\dfrac{dy}{dx}$ when $y^2 + x^2 + 8xy = 4$. ***(3 marks)***

A4 Calculate $(1, 2, 3) \times ((1, 1, 1) + (-1, -2, -3))$. ***(3 marks)***

A5 Evaluate $\displaystyle\int_1^e \frac{(\ln x)^2}{x}\, dx$. ***(3 marks)***

A6 Evaluate $\displaystyle\int \frac{dx}{\sqrt{x^2+2x}}$. *(3 marks)*

A7 Express $\displaystyle\frac{x^4+7}{(2x-1)^3(x^2+x+7)}$ in partial fractions but **do not** attempt to evaluate any of the constants involved. *(3 marks)*

A8 Find the inverse Laplace transform of the function $\displaystyle\frac{2s+5}{s^2+6s+10}$. *(3 marks)*

A9 Solve the differential equation $(1+x^2)\dfrac{dy}{dx} = x^2(1+y^2)$ given that $y(0)=0$. *(3 marks)*

A10 If $A = \begin{bmatrix} 3 & 2 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$ evaluate any of $AB,\ A^TB,\ AB^T,\ A^TB^T$ which exist. *(3 marks)*

Section B

B1 Solve $\cosh^2(x)\sinh(x) = 0$ for x a real number.
Express $\cosh^2(x)\sinh(x)$ in the form $a\sinh(nx) + b\sinh(mx)$.

B2 Solve the equation $z^3 = 1+j$ where z is a complex number and sketch the solutions on an Argand diagram.

B3 Let $r = r(x,y)$ be the distance from the origin of the point (x,y) and set $z = \ln(r)$. Find $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2}$.

B4 Evaluate each of the following integrals:

$$\int x^2 \sin 2x\, dx \quad \text{and} \quad \int \sin^2 x \cos^3 x\, dx.$$

B5 Find the general solution to the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = \sin \omega t$$

where ω is a constant.
Show that for large t, the amplitude of x is approximately $(\omega^4 - 10\omega^2 + 169)^{-1/2}$.
What value of ω maximises the amplitude?

B6 Devise a Newton-Raphson iterative scheme to solve the equation

$$x^3 + 2x^2 + x = 18.$$

Verify that $x = 2$ is a root of this equation and that the initial guess of $x = -2$ produces this root immediately.
Give a sketch to illustrate this result.

B7 Find the values of α for which the system of equations

$$\begin{aligned} x + 3y + 2z &= 1 \\ -2x + \alpha y + z &= 1 \\ 3x + 2y + \alpha z &= 0 \end{aligned}$$

has

(a) a unique solution,
(b) infinitely many solutions,
(c) no solutions.

In case (b) (and **only** for this case) find all of the solutions.

End of Question Paper

Formula Sheet for MAS156

Trigonometry

$\cos^2\theta = (1+\cos 2\theta)/2$
$\sin^2\theta = (1-\cos 2\theta)/2$
$\sin 2\theta = 2\sin\theta\cos\theta$

Hyperbolic Functions

$\cosh^2\theta = (1+\cosh 2\theta)/2$
$\sinh^2\theta = -(1-\cosh 2\theta)/2$
$\sinh 2\theta = 2\sinh\theta\cosh\theta$

Binomial theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \binom{n}{r}x^r + \ldots$$

where $\displaystyle\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$.
If n is a positive integer then the series terminates and the result is true for all x, otherwise, the series is infinite and only converges for $|x| < 1$.

Function	**Derivative**
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}$
$\cos^{-1}\left(\frac{x}{a}\right)$	$\frac{-1}{\sqrt{a^2-x^2}}$
$\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2+x^2}$
$\sinh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2+a^2}}$
$\cosh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2-a^2}}$
$\tanh^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2-x^2}$

Integration-by-Parts

$$\int uv'\,dx = uv - \int u'v\,dx$$

Substitution for a Rational Function of $\sin x$ and $\cos x$

If $t = \tan\left(\frac{x}{2}\right)$ then $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$ and $\frac{dx}{dt} = \frac{2}{1+t^2}$.

Taylor expansion of $f(x)$ about $x = a$

$$f(a) + (x-a)f^{(1)}(a) + \frac{(x-a)^2}{2!}f^{(2)}(a) + \ldots + \frac{(x-a)^{n-1}}{(n-1)!}f^{(n-1)}(a) + \ldots$$

Newton-Raphson formula for the root of $f(x) = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Table of Laplace transforms

$f(t)$	$F(s) = \mathcal{L}(f(t))$
t^n	$\frac{n!}{s^{n+1}}$ $(n = 0, 1, 2, \ldots)$
e^{at}	$\frac{1}{s-a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$e^{at}f(t)$	$F(s-a)$ (shift theorem)
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$