1. Verify that
\[ 10^4 \sum_{n=0}^{\infty} \left( \frac{1}{10}\right)^n = \sum_{n=1}^{100} 2n. \]

(10 marks)

2. Write down the first four terms of the Taylor series expansion of \( f(x) = e^{3x-1} \) about the point \( x = 0 \).

(10 marks)

3. Find and classify all the critical points of the function
\[ f(x, y) = x^3 + 2y^3 - 3x - 6y. \]

(10 marks)

4. Find:
   (i) \[ \int x \sin(x^2) \, dx \]

   (ii) \[ \int_{2}^{10} \frac{1}{x^2 - x} \, dx \]

(10 marks)
5 Let \( f(x, y) = x + y - 1 \) and \( D \subset \mathbb{R}^2 \) be the region bounded by the triangle with vertices \((0, 0), (1, 0), (0, 1)\). Find
\[
\int \int_D f(x, y)dA.
\]

(10 marks)

6 Use Gaussian elimination to solve the following system of equations:
\[
\begin{align*}
3x + 2y + 3z - 2w &= 1; \\
x + y + z &= 3; \\
x + 2y + z - w &= 2.
\end{align*}
\]

(10 marks)

7 Let
\[
A = \begin{pmatrix}
1 & 1 & 1 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{pmatrix}, \quad v = \begin{pmatrix}
1 \\
0 \\
1
\end{pmatrix}.
\]
Show that \( \{v, Av, A^2v\} \) is a basis for \( \mathbb{R}^3 \).

(10 marks)

8 Let \( a, b \) be real numbers. Find
\[
\begin{pmatrix}
a & b \\
b & a
\end{pmatrix}^{2014}
\]

Hint: It is easy to take powers of diagonal matrices.

(10 marks)

End of Question Paper