1 Obtain the general solution of the equation

\[ \sin 2\theta = 2 \cos \theta + \sin \theta - 1. \] (7 marks)

2 Factorise the polynomial

\[ x^3 + x^2 - x - 1. \] (2 marks)

3 Find the equations of the tangent and the normal to the curve \( y^2 = x^3 + x + 1 \) at the point \((0,1)\). (7 marks)

4 The equation

\[(x + y)^2 = 4(xy + 1)\]

reduces to two equations of parallel straight lines. Write down the equations of those straight lines.

[Hint: Expand the brackets and simplify the equation.] (6 marks)

5 Determine whether the point \((9,14)\) lies on the line given by the parametric equations:

\[ x = 1 + 2t \quad \text{and} \quad y = 2 + 3t. \] (2 marks)

6 Solve the following equation for real \(x\):

\[ 4^x + 2^x - 2 = 0. \] (6 marks)
Differentiate, with respect to $t$, the functions
(a) $\ln(2t^2 + 2), \quad (2 \text{ marks})$
(b) $\sin^3 t. \quad (2 \text{ marks})$

Find the following integral
$$\int \frac{4x + 3}{x^2 + 1} \, dx. \quad (4 \text{ marks})$$

Evaluate $\int_{-1}^{2} |x| \, dx$ where

$$|x| = \begin{cases} 
-x & x < 0, \\
x & x > 0.
\end{cases}$$

Note that the function $|x|$ is not differentiable at $x = 0. \quad (5 \text{ marks})$

Find the domains of the following functions:
(a) $\ln(1 - x), \quad (4 \text{ marks})$
(b) $\frac{1}{x^2}$.

Find the arithmetic and geometric mean of 1, 2, 32. \quad (3 \text{ marks})

(a) Showing your working clearly, find the coefficient of $x^3$ in the expansion of $(1 + x)^{27}. \quad (2 \text{ marks})$
(b) Use the binomial theorem to evaluate
$$\lim_{x \to \infty} \left[ \sqrt{x^2 + 8x + 3} - x - 3 \right]. \quad (3 \text{ marks})$$

Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by
$$\mathbf{a} = (-1, 2, 1), \quad \mathbf{b} = (1, -2, 3), \quad \mathbf{c} = (2, 0, 1).$$

(a) Verify that
$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}). \quad (6 \text{ marks})$$
(b) Verify that
$$\mathbf{b} \times (\mathbf{a} \times \mathbf{c}) = (\mathbf{b} \cdot \mathbf{c})\mathbf{a} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}. \quad (6 \text{ marks})$$
14. Prove, from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, the identity

$$\cosh x \cosh y + \sinh x \sinh y = \cosh(x+y).$$

(3 marks)

15. Evaluate

$$\int_{0}^{1} \frac{3x^2 + 2x - 7}{(x-2)(x^2 - x - 2)} \, dx.$$  

(9 marks)

16. Find the first 3 terms of the Maclaurin series for $\frac{1}{2 - e^x}$.  

(5 marks)

17. Express the complex number $z = 24 + 7i$ in polar form.  

Find the four values of $z^{1/4}$ in exponential form, and plot them on an Argand diagram.  

(4 marks)

18. A set of linear equations can be written as

$$\begin{pmatrix} -\lambda & 0 & 2 \\ 3 & 1 & \lambda - 4 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

where

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$  

Find the values of $\lambda$ for which non-zero solutions $X$ can exist.  

(4 marks)

For each of these values of $\lambda$, find the corresponding solution $X$.  

(6 marks)

End of Question Paper