The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2014–2015**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5 only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below. If you attempt *more* than three only your best three will be counted.

(a) Which problem has historian of mathematics E. T. Bell dubbed *the greatest Egyptian pyramid*, and why? Outline the problem and its history. ***(7 marks)***

(b) Give the **surnames** of the Dane **JLH**, who produced the definitive Greek text of the *Elements*, and the Cambridge academic **TLH**, who translated it into English. In which centuries did these Greek and English editions of the *Elements* appear? ***(2 marks)***

Identify the *two* propositions below, taken from **TLH**'s English edition of the *Elements*, by their Book and Proposition numbers.

($\boldsymbol{\alpha}$) *If in a triangle, the square on one of the sides equals the squares on the remaining two sides, then the angle contained by the remaining two sides of the triangle is right.*

($\boldsymbol{\beta}$) *On a given line to construct an equilateral triangle.*

What is ($\boldsymbol{\alpha}$) usually called? Comment on the nature of the *tacit* assumption that Euclid makes in proving ($\boldsymbol{\beta}$) that cannot be justified from his basic premises? For each of ($\boldsymbol{\alpha}$) and ($\boldsymbol{\beta}$), state how Euclid indicates the completion of its proof and explain your answer. ***(5 marks)***

(c) What did Scipione del Ferro, Tartaglia and Cardan discover in their study of cubic equations? What did they do with their discoveries? ***(7 marks)***

(d) State Archimedes' formula for the area of a parabolic segment S. Let P_1, P_2, P_3 be the first *three* polygons in Archimedes' exhaustion of S. How many sides does each have? Express the area of P_2 in terms of that of P_1. ***(4 marks)***

Use Archimedes' formula to show that $\int_{-a}^{a} x^2\,dx = \frac{2}{3}a^3$ $(a > 0)$. ***(3 marks)***

2 What *type* of number system did the Babylonians use? Mention *two* of its strengths and *one* of its weaknesses. What novel feature entered the system in the Seleucid period?
(5 marks)

Give *two* definitions of a *regular sexagesimal*. State the only regular sexagesimal *strictly* between 120 and 128, and find its *sexagesimal* reciprocal. ***(3 marks)***

Write the first three entries a, b, c in a row of **Plimpton 322**, the *eleventh* excepted, in terms of two *regular sexagesimals* p and q $(p > q)$. Express a in terms of b and c. Given that $b = 65$ and $c = 97$, find a correct to its *first* sexagesimal place. ***(8 marks)***

3 What do our earliest sources for Egyptian and Greek mathematics tell us about the differing attitudes of these civilizations to the study of geometry? ***(7 marks)***

State *the three classical problems of antiquity* and assess their importance in the history of mathematics. ***(9 marks)***

4 Below are titles of books by *three* British mathematicians. For each book, name its author and indicate its contents.

The Urinal of Physick

A Plane Discovery of the Whole Revelation of Saint John

A Briefe and True Report of the New Found Land of Virginia ***(4 marks)***

For the *two* of these authors who published mathematics books during their lifetimes, give the title of *one* such book, say what motivated its writing, give a reason for the choice of language in which it was written, and comment on its success. ***(6 marks)***

Suggest *three* reasons why the *third* author published *no* mathematics book during his lifetime. How was this compensated for in the decade following his death? How has a renewed interest in him shown itself in recent decades? ***(6 marks)***

5 In each of **(a)**, **(b)**, **(c)** below, $a > 0$. Use Fermat's *adequality* methods to show that:

(a) the values of x that make the function $y = x^3 - ax^2$ stationary are 0 and $\frac{2}{3}a$.
(4 marks)

(b) the length of the subtangent to the curve $y = \sqrt{x}$ at the point $(a, \sqrt{a})$ is $2a$.
(5 marks)

(c) the area under the curve $y = 1/x^2$ and above the x-axis interval $[a, \infty)$ is $1/a$.
(6 marks)

Why might Fermat have viewed his result in **(c)** above as remarkable? ***(1 mark)***

End of Question Paper