1 Attempt three of questions (a), (b), (c), (d) below. If you attempt more than three only your best three will be counted.

(a) Which problem has historian of mathematics E. T. Bell dubbed the greatest Egyptian pyramid, and why? Outline the problem and its history. (7 marks)

(b) Give the surnames of the Dane JLH, who produced the definitive Greek text of the Elements, and the Cambridge academic TLH, who translated it into English. In which centuries did these Greek and English editions of the Elements appear? (2 marks)

Identify the two propositions below, taken from TLH’s English edition of the Elements, by their Book and Proposition numbers.

(α) If in a triangle, the square on one of the sides equals the squares on the remaining two sides, then the angle contained by the remaining two sides of the triangle is right.

(β) On a given line to construct an equilateral triangle.

What is (α) usually called? Comment on the nature of the tacit assumption that Euclid makes in proving (β) that cannot be justified from his basic premises? For each of (α) and (β), state how Euclid indicates the completion of its proof and explain your answer. (5 marks)

(c) What did Scipione del Ferro, Tartaglia and Cardan discover in their study of cubic equations? What did they do with their discoveries? (7 marks)

(d) State Archimedes’ formula for the area of a parabolic segment S. Let $P_1, P_2, P_3$ be the first three polygons in Archimedes’ exhaustion of S. How many sides does each have? Express the area of $P_2$ in terms of that of $P_1$. (4 marks)

Use Archimedes’ formula to show that $\int_{-a}^{a} x^2 \, dx = \frac{2}{3}a^3 \ (a > 0)$. (3 marks)
2 What type of number system did the Babylonians use? Mention two of its strengths and one of its weaknesses. What novel feature entered the system in the Seleucid period? 

(5 marks)

Give two definitions of a regular sexagesimal. State the only regular sexagesimal strictly between 120 and 128, and find its sexagesimal reciprocal. 

(3 marks)

Write the first three entries $a, b, c$ in a row of Plimpton 322, the eleventh excepted, in terms of two regular sexagesimals $p$ and $q$ ($p > q$). Express $a$ in terms of $b$ and $c$. Given that $b = 65$ and $c = 97$, find $a$ correct to its first sexagesimal place. 

(8 marks)

3 What do our earliest sources for Egyptian and Greek mathematics tell us about the differing attitudes of these civilizations to the study of geometry? 

(7 marks)

State the three classical problems of antiquity and assess their importance in the history of mathematics. 

(9 marks)

4 Below are titles of books by three British mathematicians. For each book, name its author and indicate its contents.

The Urinal of Physick
A Plane Discovery of the Whole Revelation of Saint John
A Briefe and True Report of the New Found Land of Virginia 

(4 marks)

For the two of these authors who published mathematics books during their lifetimes, give the title of one such book, say what motivated its writing, give a reason for the choice of language in which it was written, and comment on its success. 

(6 marks)

Suggest three reasons why the third author published no mathematics book during his lifetime. How was this compensated for in the decade following his death? How has a renewed interest in him shown itself in recent decades? 

(6 marks)

5 In each of (a), (b), (c) below, $a > 0$. Use Fermat's adequality methods to show that:

(a) the values of $x$ that make the function $y = x^3 - ax^2$ stationary are 0 and $\frac{2}{3}a$. 

(4 marks)

(b) the length of the subtangent to the curve $y = \sqrt{x}$ at the point $(a, \sqrt{a})$ is $2a$. 

(5 marks)

(c) the area under the curve $y = 1/x^2$ and above the $x$-axis interval $[a, \infty)$ is $1/a$. 

(6 marks)

Why might Fermat have viewed his result in (c) above as remarkable? 

(1 mark)

End of Question Paper