SCHOOL OF MATHEMATICS AND STATISTICS  
Spring Semester 2014–2015

Mathematical Methods for Statistics  
2 hours

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations. Candidates should attempt ALL questions. The paper will be marked out of 80 and the allocation of marks is shown in brackets.

1  
(i) Compute \( \sum_{k=1}^{\infty} \frac{1}{3^k} \)

(ii) Find constants \( A \) and \( B \) so that:

\[
\frac{1}{4n^2 - 1} = \frac{A}{2n - 1} + \frac{B}{2n + 1}
\]

(iii) Compute \( \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \)  

(10 marks)

2  
Compute the derivatives of the following functions with respect to \( x \).

(i) \( r(x) = x^2 \ln(x) \)

(ii) \( s(x) = e^{x+x^{-1}} \)

(iii) \( t(x) = \frac{\cos(x)}{\sin(x)} \)

(10 marks)

3  
Find and classify all critical points of \( f(x, y) = 6x^2 + 6xy + y^2 - 2y \)  

(10 marks)
4. Compute the definite integrals

(i) \[ \int_{-2}^{2} \frac{1}{2x + 3} \, dx \]

(ii) \[ \int_{0}^{\pi/4} x \sin(2x) \, dx \] (10 marks)

5. Let \( D \) be the region bounded by the inequalities \( x \geq 0, y \geq 0, \) and \( x^2 + y^2 \leq 1. \) Find \( \iint_{D} x \, dx \, dy. \) (10 marks)

6. Use Gaussian elimination to solve the following system of equations:

\[
\begin{align*}
\quad x + 2y + 3z &= 6 \\
2x - y + z &= 0 \\
3x - 2y - z &= -2
\end{align*}
\] (10 marks)

7. Let

\[
M = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ 3 \end{pmatrix}
\]

Find:

(i) \( vM \)

(ii) \( Mw \)

(iii) \( M^{-1} \) (10 marks)

8. Let

\[
A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}
\]

(i) Find the eigenvectors of \( A \) and the associated eigenvalues.

(ii) Use your result from Part (i) to find

\[ A^{100} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \] (10 marks)

End of Question Paper