1 Find the constants $r$ and $\alpha$, with $r > 0$ and $-\pi < \alpha \leq 0$, such that

$$\cos x - \sqrt{3}\sin x = r \cos(x - \alpha).$$

(4 marks)

Hence find all solutions $x$ of the equation

$$\cos x - \sqrt{3}\sin x = \sqrt{2}.$$

(4 marks)

2 Solve the equation

$$2 \ln x - \ln(x-3) = \ln(x-6).$$

(6 marks)

3 Identify the number $d$ such that

$$\frac{4^{9/2}3^{11}}{6(3^5)^28^{3/3}} = 2^d.$$

(4 marks)

4 Simplify

$$\frac{(27x^6y^9z^6)^{1/3}}{(\sqrt[3]{3}x^{1/2}y\sqrt{z})^2}.$$

(4 marks)
Factorise \(3x^2 - 9x + 6\). \(\text{(2 marks)}\)

Given a function \(y = f(x)\) where \(f(x) = \frac{x - 1}{x - 2}\).

(i) Draw the graph of the function \(y = f(x)\). \(\text{(4 marks)}\)

(ii) Find the domain \(D\) and the range \(R\) of the function \(y = f(x)\). \(\text{(2 marks)}\)

(iii) Find the inverse \(f^{-1}(x)\) of the function \(f(x)\). \(\text{(4 marks)}\)

(iv) Find the domain \(D'\) and the range \(R'\) of the inverse function \(y = f^{-1}(x)\). \(\text{(2 marks)}\)

Verify the following identity

\[
\frac{\cos^4 x + \frac{1}{2} \sin^2(2x) + \sin^4 x}{1 + \tan^2 x} = \cos^2 x.
\]

\(\text{(5 marks)}\)

Let \(y = e^{x^2 + \sin x}\). Find \(\frac{dy}{dx}\) and \(\frac{d^2 y}{dx^2}\). \(\text{(4 marks)}\)

Solve the following equation for real \(x\):

\[9^x - 3^x - 6 = 0.\]

\(\text{(5 marks)}\)

(i) Showing your working clearly, find the coefficient of \(x^3\) in the expansion of \((1 + x)^{12}\). \(\text{(2 marks)}\)

(ii) Use the binomial theorem to evaluate

\[
\lim_{x \to \infty} (\sqrt{x^2 + 6x - 4} - x - 2).
\]

\(\text{(3 marks)}\)

(i) Show that the vectors \(\mathbf{u} = (3, 0, 3)\) and \(\mathbf{v} = (2, 12, -2)\) are perpendicular. \(\text{(3 marks)}\)

(ii) A plane passes through the points \(\mathbf{a} = (0, 0, 1), \mathbf{b} = (1, 1, -1)\) and \(\mathbf{c} = (2, -2, 1)\). Find the Cartesian equation of the plane. \(\text{(6 marks)}\)
12. Prove, from the definitions of sinh \( x \) and \( \cosh x \), the identity
\[
2 \sinh x \cosh x = \sinh 2x.
\] (3 marks)

13. Evaluate
\[
\int \coth x \, dx.
\] (4 marks)

14. Evaluate
\[
\int \frac{2x + 1}{\sqrt{x^2 + 4}} \, dx.
\] (8 marks)

15. Find the Maclaurin series for \( f(x) = e^{x^2 + 1} \), as far as the term in \( x^3 \). (7 marks)

16. Complex numbers \( z_1 \) and \( z_2 \) are defined by
\[
z_1 = 1 + i, \quad z_2 = 2 - i.
\]
Find, in the form \( a + bi \) where \( a \) and \( b \) are real,

(i) \( z_1^3 \), (2 marks)

(ii) \( \frac{z_2}{2z_1 + z_2} \). (3 marks)

17. Solve the system of linear equations
\[
\begin{align*}
x + 2y + z &= 3, \\
x + 3y + 2z &= 4, \\
2x + 5y + 3z &= 7.
\end{align*}
\] (5 marks)

18. Find the volume \( V \) of the parallelepiped determined by the vectors \( \mathbf{a} = (1, 1, 1) \), \( \mathbf{b} = (1, 1, 0) \) and \( \mathbf{c} = (2, 1, 1) \). (4 marks)

End of Question Paper