The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2015–2016**

Mathematical Methods for Statistics **2 hours**

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.
*Candidates should attempt **ALL** questions.*
The paper will be marked out of 80 and the allocation of marks is shown in brackets.

1 (i) Compute $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2^k}$

(ii) Find constants A and B so that:

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

(iii) Compute $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ **(10 marks)**

2 Compute the derivatives of the following functions with respect to x.

(i) $r(x) = \tan(3x^2 + 1)$

(ii) $s(x) = e^{2x} \ln x$

(iii) $t(x) = \dfrac{x}{x+1}$

(iv) $w(x) = x^x$

(10 marks)

3 Show that $x = 2$ and $y = 4$ is a critical point of

$$f(x, y) = 2xy + \frac{64}{y} + \frac{32}{x},$$

and classify this critical point. **(10 marks)**

4 Compute the definite integrals

(i) $\displaystyle\int_0^{\pi/2} \sin 2x \cos x dx$

(ii) $\displaystyle\int_0^{\infty} \frac{1}{1+x^2} dx$ **(10 marks)**

5 Let S be the region $\{(x, y) : 1 \leq x \leq 2, 0 \leq y \leq 1\}$. Calculate the double integrals

(i) $\displaystyle\int\int_S (x^2 - 3y^2)\, dx\, dy.$

(ii) $\displaystyle\int\int_S \frac{1}{(x+y^2)^2}\, dy\, dx.$ **(10 marks)**

6 Let

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 \\ 4 & -1 \end{pmatrix}.$$

Find:

(i) AB

(ii) BA

(iii) A^{-1}

7 Let **v**, **w**, **a**, and **b** be vectors

$$\mathbf{v} = (3, -3, 1), \quad \mathbf{w} = (4, 9, 2), \quad \mathbf{a} = (2, 1, 4, -1), \quad \mathbf{b} = (4, -1, 0, 2).$$

(i) Calculate $\mathbf{v} \times \mathbf{w}$.

(ii) Calculate $\mathbf{v} \cdot \mathbf{w}$.

(iii) Calculate $\mathbf{a} \cdot \mathbf{b}$ and hence find the angle between **a** and **b** in radians to 2 decimal places.

(10 marks)

8 Let

$$A = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

Find the eigenvectors of A and the associated eigenvalues.

(10 marks)

End of Question Paper