The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2016-2017**

CORE FOUNDATION MATHEMATICS **3 hours**

*Attempt **all** questions. The allocation of marks is shown in brackets. Total marks: 100.*
NO CALCULATORS ALLOWED.

1 (i) The temperature of a liquid, T (measured in Celsius) as a function of time, $t \geqslant 0$ (measured in minutes) obeys the equation

$$T(t) = 128 \cdot 2^{-t}.$$

Determine how many minutes it will take for the temperature of the liquid to reach 32 Celsius. ***(4 marks)***

(ii) (a) Expand and simplify

$$L = \frac{1}{2}\left(\ln 2 + \ln\left(\frac{eB^5}{B^3 2e^{2C}}\right)\right).$$

(b) Using your answer from part **1**(ii)(a) above, show that if $B = e^{2C}$ and $C = \frac{1}{2}$, then $L = 1$. ***(6 marks)***

2 (i) Without making use of Pascal's triangle, find the first three terms when $(1+3z)^6$ is expanded in ascending powers of z, showing *all* your working. ***(6 marks)***

(ii) Find

(a) $$\sum_{N=1}^{20}(5N+2).$$

(b) $$\sum_{\alpha=1}^{\infty}\left(\frac{1}{8}\right)^{\alpha}.$$ ***(4 marks)***

(iii) A sample of mould is kept in a container. At the end of the first day, the mould occupies $\frac{1}{8}$ of the container. At the end of each of the following two days, the mould occupies twice as much space as it did at the end of the previous day.

(a) What *additional* fraction of the container will the mould have occupied during *each* of the second and third days?

(b) On the fourth day, the mould grows an amount equal to half its area at the start of the day. On each subsequent day, the mould grows an amount equal to 50% of the previous day's growth. Assuming that this rate does not change, what percentage of the container would be occupied by the mould as the number of days, $d \to \infty$? Will the mould ever spread to fill the container completely? ***(4 marks)***

3 (i) (a) Write the formula for the probability, P of a random event, E occurring.

(b) Write the probability of the event, A of rolling an unbiased die and obtaining the number ‘4’.

(c) Write the probability for the event in part **3**(i)(b) above, *not* occurring.

(d) The probability of the event B of rolling an unbiased tetrahedral die and obtaining the number ‘3’ is given by the equality

$$\mathrm{P}(B) = 1.$$

Write down the possibility space of the event B. ***(4 marks)***

(ii) Given the list:

$$\{1, 1, 3, 5, 8, 10, 13, 21, 34\},$$

let:
X be the random selection of an odd number;
Y be the random selection of an even number; and,
Z be the random selection of an prime number.

(a) Write the probability of randomly selecting either an even number or a prime number.

(b) Suppose we simultaneously select a random number from the above list and roll an unbiased die. Let S be the event of randomly selecting the number ‘5’ from the list above. Let T be the event of obtaining the number ‘5’ from rolling the die. Write down the probability of both S and T occurring. ***(4 marks)***

(iii) The probability distribution of a discrete random variable, R is as follows:

r	1	2	3	4
P_r	0.125	0.500	P_3	0.125

where P_r denotes the probability that $R = r$. Find

(a) P_3.

(b) $\mathrm{E}(R)$. ***(4 marks)***

4 (i) Let $h(y) = \cos y + \sqrt{3}\sin y$.

(a) Write $h(y)$ in the form

$$h(y) = K\sin(y+\theta), \qquad 0 < \theta < \frac{\pi}{2}.$$

(b) Determine the minimum value of $h(y)$ and the values of y for which this minimum value occurs.

(c) Determine the maximum value of $H(y) = h(y) - 3$ and the values of y for which this maximum value occurs. ***(8 marks)***

(ii) Using the identity

$$\cos^2\left(\frac{\beta}{2}\right) + \sin^2\left(\frac{\beta}{2}\right) = 1,$$

prove that

$$\operatorname{cosec}\beta = \frac{1}{2}\left(\tan\left(\frac{\beta}{2}\right) + \cot\left(\frac{\beta}{2}\right)\right), \qquad 0 < \beta < \pi.$$

(6 marks)

5 (i) Find the derivative of $f(x) = 4 - x - x^2$ by using *first principles.*

(ii) Differentiate $y = x^5 + \dfrac{2}{x^{\frac{1}{3}}} - \sqrt[7]{x} + 9e$.

(iii) Differentiate $y = \tan^{-1}(e^x)$.

(iv) Differentiate $y = (\sin x)(3x^4 - 1)$.

(v) Differentiate $y = \dfrac{x}{\tan x}$.

(vi) Find $\dfrac{dy}{dx}$ in terms of t when $y = \dfrac{t}{\ln t}$ and $x = \dfrac{t}{t^2+1}$.

(vii) Find $\dfrac{dy}{dx}$ if $y^2 + e^y = \ln x - 3xy + x$. ***(14 marks)***

6 (i) Find $\int \left(x + \frac{4}{x^{\frac{3}{4}}} + \frac{1}{x} + 7 \right) dx$.

(ii) Find $\int (12x^3 + 9x^2)e^{x^4+x^3}\, dx$.

(iii) Find $\int x \sin x \, dx$.

(iv) Find $\int_0^{2\sqrt{3}} \frac{1}{(\frac{1}{2}x)^2 + 1}\, dx$. ***(7 marks)***

7 Let $y = f(x) = \frac{x}{x^2 - 4}$.

(i) Sketch the graph of $y = f(x) = \frac{x}{x^2 - 4}$. ***(8 marks)***

(ii) Find the *area* enclosed by $y = \frac{x}{x^2 - 4}$, the x-axis, and the lines $x = -1$ and $x = 1$. ***(5 marks)***

8 Find the stationary points of $y = f(x) = \frac{1}{8}x^8 - \frac{5}{6}x^6 + x^4$ and determine their nature. ***(10 marks)***

9 Find the equation of the tangent and the normal to the curve $y = f(x) = x^3 + \ln x$ at the point $(1, 1)$. ***(6 marks)***

End of Question Paper