



The  
University  
Of  
Sheffield.

**MAS004**

**SCHOOL OF MATHEMATICS AND STATISTICS**

**Spring Semester  
2016–2017**

**Further Foundation Mathematics**

**2 hours**

*Answer all questions. Total marks 80.*

*You should justify your answers carefully unless the question states otherwise.*

1 (i) Let  $z = \frac{1}{2}(1 - \sqrt{3}i)$ .

(a) Indicate 1,  $z$  and  $z^{-1}$  all on the same Argand diagram. (3 marks)

(b) By cubing  $z$ , algebraically show that  $z^{3/2} = i$  and hence that

$$z^{15/2} = i.$$

(4 marks)

(c) What is  $z^{51}$ ?

(2 marks)

(ii) (a) Describe in geometric terms the locus of points  $C$  satisfying

$$x^2 + 4x + y^2 - 6y = 0.$$

(4 marks)

(b) Show that the line  $3y = 2x$  is a tangent line to this locus and state the point of intersection. (7 marks)

**2** In this question, let  $\mathbf{u} = 5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{v} = \mathbf{j} - 2\mathbf{k}$ , and  $\mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ .

(i) (a) Evaluate  $\mathbf{u} \times \mathbf{v}$ ,  $\mathbf{u} \cdot \mathbf{w}$  and  $\mathbf{v} \cdot \mathbf{w}$ .

Hence evaluate  $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$  and  $\mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{u}(\mathbf{v} \cdot \mathbf{w})$ .  
(7 marks)

(b) Find the value of  $\lambda$  such that  $\mathbf{u} + \lambda\mathbf{v}$  is perpendicular to  $\mathbf{w}$ .  
(2 marks)

(c) Find the value of  $\mu$  such that  $\mathbf{u} + \mu\mathbf{v}$  is parallel to  $\mathbf{w}$ . (3 marks)

(ii) Consider the plane whose points have coordinate vector  $\mathbf{x}$  satisfying the equation

$$\mathbf{x} = \mathbf{u} + a\mathbf{v} + b\mathbf{w}.$$

Which of the following points lie on the plane? State the required values of  $a$  and  $b$  where appropriate.

(a)  $4\mathbf{i} + \mathbf{j} + \mathbf{k}$ ;

(b)  $7\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$ ;

(c)  $3\mathbf{i} + 2\mathbf{j}$ . (8 marks)

**3** (i) Sketch a graph of the function

$$f(x) = \frac{4}{1 + 2x} - \frac{2}{(2 - x)}.$$

List the points where it is discontinuous. (4 marks)

(ii) (a) Evaluate  $\prod_{p=2}^6 (2p - 3)$ . (4 marks)

(b) If  $m = 5$ , evaluate  $\frac{1}{2} \sum_{n=1}^m (12mn - 2)$ . (4 marks)

(iii) (a) Showing your working carefully, calculate the terms of degree up to and including  $x^3$  in the binomial expansion of  $(3 - 2x)^{1/4}$ . (4 marks)

(b) By squaring the expansion just obtained, find the terms of degree up to and including  $x^3$  of a Maclaurin series for  $(3 - 2x)^{1/2}$ . (4 marks)

- 4 (i) (a) Find, by the method of separation of variables or otherwise, the general solution to the differential equation

$$ye^{-x} \frac{dy}{dx} = 1.$$

*(7 marks)*

- (b) Hence find a solution which has  $y = 2$  when  $x = 0$ . *(3 marks)*

- (ii) (a) Find the general solution to the differential equation

$$3 \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 6y = 0.$$

*(6 marks)*

- (b) Hence find a solution which has  $y = 1$  and  $\frac{dy}{dx} = 0$  when  $x = 0$ . *(4 marks)*

**End of Question Paper**