The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2016–2017**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5 only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below. If you attempt *more* than three only your best three will be counted.

(a) What is known about the Egyptians' mathematical knowledge of the square pyramid? A frustum of such a pyramid has top edge a, bottom edge b and height h, all in cubits. Express the volume of the frustum and the seqt of its faces in terms of a, b, h with relevant units. Speculate on their knowledge of the volume of a square pyramid. ***(7 marks)***

(b) Place *Eudemus*, *Plato*, and *Thales* in chronological order. For each, write a few sentences on their place in the history of Greek mathematics. ***(7 marks)***

(c) What light does the history of the cubic equation shed on *the mathematical scene* in Italy during the first half of the sixteenth century? ***(7 marks)***

(d) What is a palimpsest? Which lost work was *famously* found on a palimpsest in 1906?

You are now asked to work through a proof of Proposition 4 from that work, giving the *volume of a paraboloid of revolution.*

Let $O = (0, 0)$, $U = (h^2, -h)$, $V = (0, -h)$, $W = (0, h)$, $X = (h^2, h)$, where $h > 0$. Then O, U, X lie on the parabola $x = y^2$. Let C be the cylinder formed by rotating rectangle $UVWX$ about the x-axis and let P be the paraboloid of revolution formed by rotating parabolic segment UOX about the x-axis. Show the vertical circular slice of P at $(x, 0)$, placed with centre at $(-h^2, 0)$, balances the vertical slice of C at $(x, 0)$ about fulcrum O $(0 \leqslant x \leqslant h^2)$. Hence express the volume of P as a fraction of that of C. ***(7 marks)***

2 **Plimpton 322** displays n columns of Babylonian numbers arranged in m rows. Give the values of m and n. Where is it currently exhibited? Name the period from which it dates, the script that it bears, and the work in which it was first analyzed. ***(6 marks)***

The second and third entries, a and c say, on a row of **Plimpton 322** are, respectively, the shortest and longest sides of a right-angled triangle with integer sides. Let b be the third side of this triangle, and A its smallest angle. For any row except the eleventh, write a, b, c, A and the first entry on the row, in terms of two *regular* sexagesimals p and q. Given that on the thirteenth row, a is 161 and c is 289, find p, q, b, A (the last to the nearest degree), and the first entry on the row to *one* sexagesimal place. How are the rows on the tablet arranged? ***(10 marks)***

3 Outline the structure and the contents of **Book I** of Euclid's *Elements*, illustrating your answer with specific examples. ***(9 marks)***

Comment on the following extracts relating to the *Elements*.

(a) *Give him a coin, for he must profit from what he learns.*

(b) He studied the first six books of Euclid from the time he entered Congress.

(c) His demonstrations require axioms of which he is unaware. A valid proof retains its demonstrative form when no figure is drawn, but some of Euclid's early proofs fail before this test. ***(7 marks)***

4 Describe Robert Recorde's contribution to British mathematics in the sixteenth century. To what do you attribute his success as an author? ***(10 marks)***

For each of the *two* extracts below, one not appearing in the course, identify its source and indicate the devices used by Recorde in it to engage his reader.

(i) Master But there are more benefits to arithmetic. Why are accountants so well rewarded, geometers so highly respected, and astronomers so knowledgeable? It is because by numbers, they have made discoveries, impossible without them.

Scholar If men by numbers great things achieve, arithmetic is subtler than I thought.

Master If numbers were crude, it would not be much used in communication. Answer me, without numbers. How old are you? How many days in a week, weeks in a year?

Scholar Um. Um. Um.

Master All your answers are ums. How many miles to London?

Scholar A pocket full of ums.

Master If number be lacking, it maketh men dumb, many an answer no longer than um.

(ii) *Since merchants by ships great riches do win,*
I will with good reason at their seat begin.
The shippes on the sea with sail and with oar,
Were first found and still made by geometry's law. ***(6 marks)***

5 By mid-seventeenth century a profusion of techniques for finding areas under curves and tangents to them were known. What, then, remained for the calculus to be *invented*? What work first presented an account of the *newly invented* calculus? When was this work published? ***(5 marks)***

Let the tangent T to the curve $y = f(x)$ at the point $(a, f(a))$ meet the x-axis at the point $(a - t, 0)$, where $t > 0$. How can T be constructed once t is known? Obtain the *adequality*

$$\frac{f(a)}{t} \sim \frac{f(a+E)}{t+E} \quad (small\ E > 0).$$ ***(3 marks)***

How *might* Fermat have used the adequality to show that $t = \frac{2}{3}a$, when $y = \frac{2}{3}x^{\frac{3}{2}}$ and $a > 0$? Deduce dy/dx in this case. ***(8 marks)***

End of Question Paper