



The
University
Of
Sheffield.

MAS5050

SCHOOL OF MATHEMATICS AND STATISTICS

**Spring Semester
2016–2017**

Mathematical Methods for Statistics

2 hours

RESTRICTED OPEN BOOK EXAMINATION

Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.

*Candidates should attempt **ALL** questions.*

The paper will be marked out of 80 and the allocation of marks is shown in brackets.

**Please leave this exam paper on your desk
Do not remove it from the hall**

Registration number from U-Card (9 digits)
to be completed by student

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1 (i) Compute $\sum_{k=1}^{\infty} \frac{1}{5^k}$

(ii) Find the constants A and B so that:

$$\frac{1}{n^2 + 4n + 3} = \frac{A}{n + 1} + \frac{B}{n + 3}$$

(iii) Compute $\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n + 3}$ **(10 marks)**

2 Compute the derivatives of the following functions with respect to x .

(i) $r(x) = \sin(2x^2 + 3)$

(ii) $s(x) = e^{9x} \tan x$

(iii) $t(x) = \frac{\ln x}{x^2}$

(10 marks)

3 Find and classify the stationary points of the function

$$f(x, y) = x^3 + 15x^2 - 20y^2 + 10.$$

(10 marks)

4 (i) By making a suitable substitution determine the indefinite integral

$$\int (1 + \sin x)^7 \cos x \, dx.$$

(ii) Compute the definite integral

$$\int_0^1 x e^{5x} \, dx.$$

You may leave your final answer in terms of e .

(10 marks)

5 Let S be the region $\{(x, y) : 0 \leq x \leq 1, 1 \leq y \leq 2\}$. Calculate the double integrals

(i) $\int \int_S (x^2 + xy^3) \, dx \, dy.$

(ii) $\int \int_S \frac{x}{y^2} \, dy \, dx.$ **(10 marks)**

6 Let

$$A = \begin{pmatrix} 8 & 1 \\ 5 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 6 \\ 2 & 4 \end{pmatrix}.$$

- (i) Calculate AB .
- (ii) Calculate BA .
- (iii) Find A^{-1} and verify that your answer is correct by calculating $A^{-1}A$ and AA^{-1} . **(10 marks)**

7 Use Gaussian elimination to solve the following system of equations:

$$\begin{aligned} x + 2y + z &= 8 \\ 8x + y - z &= 37 \\ 3x - 3y + 2z &= -1 \end{aligned}$$

(10 marks)

8 (i) Let \mathbf{a} and \mathbf{b} be vectors such that

$$\mathbf{a} = (6, -2, 4), \quad \mathbf{b} = (1, 0, 4).$$

- (a) Calculate $\mathbf{a} \cdot \mathbf{b}$.
 - (b) Calculate $\mathbf{a} \times \mathbf{b}$.
- (ii) Let

$$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}.$$

Find the eigenvalues of A .

(10 marks)

End of Question Paper