The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **June 2017**

MAS5051 Probability and Probability Distributions **2 hours**

RESTRICTED OPEN BOOK EXAMINATION.

Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.

*Candidates should attempt **All** questions.*

The maximum marks for the various parts of the questions are indicated.

The paper will be marked out of 80.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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1 Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{2}{x^2} & 1 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

(i) Verify that $f(x)$ is indeed a valid probability density function. ***(3 marks)***

(ii) Find the distribution function of X. ***(5 marks)***

(iii) Calculate $P(X > 1.5|X > 1.25)$. ***(3 marks)***

(iv) Let $Y = \ln(X)$, where ln denotes the natural logarithm. Find the probability density function of Y. ***(5 marks)***

2 Three balls are drawn, one at a time, without replacement from an urn containing six black balls and four white balls.

(i) Describe an appropriate sample space to use for this experiment. ***(3 marks)***

(ii) Let X be the number of black balls drawn from the urn.

(a) Calculate the probability mass function of X. ***(7 marks)***

(b) Find the mean and variance of X. ***(7 marks)***

3 Let R be the region defined by $R = \left\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \frac{1}{2}\right\}$, and let X and Y be random variables with joint probability density function given by

$$f_{X,Y}(x, y) = \begin{cases} k(x + 2y) & (x, y) \in R \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the value of k. ***(4 marks)***

(ii) Find the marginal probability density functions of X and Y. ***(5 marks)***

(iii) Let $x \in (0, 1)$. Find the conditional probability density function of Y, given $X = x$. ***(4 marks)***

(iv) Calculate $\mathbb{E}[Y|X = x]$. ***(4 marks)***

4 A mail order company receives telephone orders at a constant rate of three per hour.

(i) Explain why the number of telephone orders in an hour might be assumed to be a Poisson distribution with parameter $\lambda = 3$. ***(3 marks)***

(ii) Assuming a Poisson distribution with parameter $\lambda = 3$, what is the probability that the number of telephone orders is more than one in an hour? ***(3 marks)***

(iii) Let S be the total number of telephone orders in a 75 hour period. Assuming that the number of orders in each hour is independent, what is the expectation and variance of S? ***(4 marks)***

(iv) Use Chebyshev's inequality to give a lower bound for $P(200 \leq S \leq 250)$. ***(4 marks)***

(v) Explain carefully how to use a Normal approximation to find $P(200 \leq S \leq 250)$. You may give your final answer in terms of the standard normal distribution function Φ. ***(5 marks)***

5 A factory uses a diagnostic test to determine whether a part is defective or not. This test has a 0.9 probability of giving a correct result when applied to a defective part and a 0.05 probability of giving an incorrect result when applied to a non-defective part. It is believed that one out of every thousand parts will be defective.

(i) Calculate the posterior probability that a part is defective if the test says it is defective. ***(5 marks)***

(ii) Calculate the posterior probability that a part is non-defective if the test says it is non-defective. ***(3 marks)***

(iii) Calculate the probability that a part is misdiagnosed. ***(3 marks)***

End of Question Paper