The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Autumn Semester 2017–18**

Game Theory **2 hours and 30 minutes**

Attempt all the questions. The allocation of marks is shown in brackets.

Please leave this exam paper on your desk
Do not remove it from the hall

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to be completed by student

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1 (i) Alice and Bob play a game given in strategic form as follows:

	I	II	III
A	3, 2	9, 1	1, 1
B	2, 6	10, 5	0, 5
C	1, 1	11, 0	2, 3

(a) Find all weakly dominated strategies and all strictly dominated strategies. ***(2 marks)***

(b) Eliminate iteratively all strictly dominated strategies. ***(2 marks)***

(c) Find all pure-strategy and all mixed-strategy Nash equilibria of this game. ***(8 marks)***

(ii) In Freedonia electricity is produced by $n \geq 1$ independent power stations, each capable of producing unlimited amounts of electricity. The daily price p of 1 GWh of electricity is a function of the total amount q of electricity produced by all power stations that day (in units of GWh) and is given by $p(q) = 50 - q/1000$ pounds per GWh. The cost of producing 1 GWh of electricity is £20 per GWh for all producers.

(a) Find a production profile which results in a Nash equilibrium. ***(8 marks)***

(b) Find the price of electricity at the Nash equilibrium as a function of n and discuss the advantage for consumers of perfect competition, i.e., having a large number of suppliers. (You may assume that the Nash equilibrium you found in (a) is unique.) ***(5 marks)***

2 (i) In a remote village with $n > 1$ inhabitants there is nothing to do in the evening but to meditate on the top of a nearby mountain. Meditation yields a utility of 10 units, but only one person can comfortably meditate at the top of the mountain: if two or more people climb the mountain, they get no utility. Climbing the mountain is difficult and it costs 3 units of utility. Staying at home yields no utility.

(a) Find all pure Nash equilibria of this game. Justify your answer. ***(5 marks)***

(b) Find a symmetric mixed-strategy Nash equilibrium of this game, i.e., find a mixed strategy which, if played by all players, yields a Nash equilibrium. ***(8 marks)***

(ii) Alice and Bob face the following game

	l	r
U	0, 0	1, -1
D	1, 1	0, 3

and choose to negotiate an outcome, with the knowledge that, if they fail to strike a deal, expected utilities of 1/2 for Alice and 1/3 for Bob will be imposed.

(a) Sketch the cooperative payoff region of the game. ***(4 marks)***

(b) Describe parametrically the payoffs that satisfy the *Individual Rationality* and *Pareto Optimality* conditions. ***(3 marks)***

(c) Find the Nash Bargain of this setup. ***(5 marks)***

3 (i) Alice manufactures microprocessors and Bob manufactures computers using these components. Bob can buy Alice's microprocessors or produce his own. Alice has the option of launching an ad campaign promoting her products, or not doing so. She also has the choice between charging Bob a high price or a low price.

In the event that Alice advertises and Bob buys from her, the profits for Alice and Bob are £15,000,000 and £12,000,000, respectively, if she charges a high price, and £12,000,000 and £15,000,000, respectively, if she charges a low price.

In the event that Alice does not advertise and Bob buys from her, the profits for Alice and Bob are £12,000,000 and £8,000,000, respectively, if she charges a high price, and £10,000,000 and £10,000,000, respectively, if she charges a low price.

Regardless of whether Alice decided to launch the ad campaign, if Bob decided to manufacture his own microprocessors, Alice's profits will be nil and Bob's will be £9,000,000.

(a) Describe this game using a tree, carefully labelling all its components. ***(7 marks)***

(b) Solve this game using backward induction. ***(5 marks)***

(c) List all the subgames of this game. ***(5 marks)***

(d) Describe the game in strategic form, find all its pure-strategy Nash equilibria and indicate which of these is subgame perfect. ***(8 marks)***

4 (i) Consider a 2-player game given in strategic form as (S, T, u_1, u_2).

(a) Define the *minimax values* of both players. ***(2 marks)***

(b) Define the *cooperative payoff region* of the game. ***(2 marks)***

(ii) Consider the 2-person game G given in tabular form as follows

	A	B
I	1, 1	5, 0
II	0, 5	1, 1

(a) Describe the outcome of this game when played once. If the game were repeated $n > 0$ times and both players knew that the game is being repeated n times, what would be the average payoffs per round for both players? Justify your answer. ***(4 marks)***

(b) Show that the point $(2, 2)$ is in the cooperative payoff region of G. ***(2 marks)***

(c) Consider now the game G^∞ which consists of playing G repeatedly, and where the payoffs of the infinite game are the average payoffs. Describe, without proof, a Nash equilibrium that results in an average payoff of 2 for both players. ***(6 marks)***

(iii) Alice is the only manufacturer of quantum computers and Bob is considering entering that market. There are two ways to manufacture quantum computers: one is fast and the other is slow. Bob does not know Alice's manufacturing process, but he believes that the probability of Alice using the fast process is 20%. Alice can choose between charging a high price or a low price for her product, and independently Bob needs to decide whether or not to compete with Alice. The profits in billions of pounds for Alice and Bob in all possible scenarios are given in the following tables.

Fast Alice

	Compete	Not Compete
High Price	2, -1	4, 0
Low Price	1, -2	3, 0

Slow Alice

	Compete	Not Compete
High Price	2, 3	3, 0
Low Price	0, 2	1, 0

(a) Model this as a Bayesian game. ***(3 marks)***

(b) Find a Bayes-Nash equilibrium of this game. ***(6 marks)***

End of Question Paper