



The  
University  
Of  
Sheffield.

**MAS004**

**SCHOOL OF MATHEMATICS AND STATISTICS**

**Spring Semester  
2017–2018**

**Further Foundation Mathematics**

**2 hours**

*Answer all questions. Total marks 80.*

*You should justify your answers carefully unless the question states otherwise.*

1 (i) Write the following in the form  $z = x + iy$ ;

(a)

$$z = \frac{2 + 3i}{1 + 4i},$$

*(2 marks)*

(b)

$$z = (3 + 4i)^2(1 - 2i).$$

*(2 marks)*

(ii) For  $z = -(2 + \sqrt{2}i)$ , find  $z^2, z, z^0, z^{-1}$  and  $z^{-2}$  in the form of  $x + iy$ .

*(5 marks)*

(iii) Sketch all the points satisfying  $|z - 1| = 2$  for  $z \in \mathbb{C}$ .

*(4 marks)*

(iv) (a) Sketch  $x^2 - 6x + y^2 - 4y + 9 = 0$ .

*(4 marks)*

(b) Determine if the line  $y = 2x$  is a tangent line to the equation in part

(a).

*(3 marks)*

- 2 (i) Show that for any non-zero vectors  $\mathbf{a}$  and  $\mathbf{b}$

$$\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0.$$

(5 marks)

- (ii) Find the equation of the lines passing through the following pairs of points:

(a)  $(0, 0, -1)$  and  $(1, 5, 1)$ ; (3 marks)

(b)  $(1, 1, 1)$  and  $(2, 1, 3)$ ; (3 marks)

(c)  $(5, 3, 7)$  and  $(9, 1, 17)$ . (3 marks)

- (iii) On which of the above lines (i.e. from part ii) do the following points lie:

(a)  $\frac{1}{5}(1, 5, -3)$ ; (2 marks)

(b)  $(1, 5, 1)$ ; (2 marks)

(c)  $(3, 2, 3)$ . (2 marks)

- 3 (i) Find all solutions to the following

(a)  $\sum_{n=0}^3 (n^2 - n)x^n = 0$ . (3 marks)

- (ii) Find

(a)  $\prod_{n=2}^5 2^n$ , (2 marks)

(b)  $\sum_{n=3}^7 \frac{n}{k}$  where  $k = \frac{3}{2}$ , (2 marks)

(c)  $\prod_{n=1}^4 n^3$ . (2 marks)

- (iii) (a) Find the Maclaurin series of  $y = (1 + x^2)^{-1}$  up to terms in  $x^4$ . (9 marks)

(b) Compare your result for part (a) with the exact value of  $y$  for  $x = 0.1$ . (2 marks)

- 4 (i) (a) Find, by the method of separation of variables, or otherwise, the general solution to the differential equation

$$\frac{dy}{dx} = 4xy^2 e^{-x^2}.$$

*(7 marks)*

- (b) Hence find a solution which has  $y = 1$  when  $x = 0$ . *(3 marks)*

- (ii) (a) Find the general solution to the differential equation

$$4\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 3y = 0.$$

*(6 marks)*

- (b) Hence find a solution which has  $y = 1$  and  $\frac{dy}{dx} = 0$  when  $x = 0$ .

*(4 marks)*

**End of Question Paper**